MATHEMATICS

(See also Applied Mathematics.)

Director of undergraduate studies: Andrew Casson, 216 LOM, 432-7056, andrew.casson@yale.edu; math.yale.edu

FACULTY OF THE DEPARTMENT OF MATHEMATICS

Professors Donald Brown, Andrew Casson, Ronald Coifman, Igor Frenkel, Howard Garland, Alexander Goncharov, Roger Howe, Peter Jones, Mikhail Kapranov, Gregory Margulis, Yair Minsky, Vincent Moncrief, Hee Oh, David Pollard, Vladimir Rokhlin, Peter Schultheiss (Emeritus), Van Vu, Gregg Zuckerman

Assistant Professors Amanda Folsom, Alexander Kontorovich, Sam Payne

J. W. Gibbs Assistant Professors Dustin Cartwright, Kim Dang, Swarnendu Datta, Yen Quang Do, Steven Frankel, Daniel Fresen, Asaf Hadari, Jiuzu Hong, Nathan Kaplan, Andrei Osipov, Ronen Talmon, Zhiren Wang, Peng Zhao

Adjunct Professors Michael Frame, Gil Kalai, Alex Lubotzky

Lecturers Marketa Havlickova, Anna Lachowska, James Rolf

Operations Research Faculty Eric Denardo

Statistics Faculty Andrew Barron, Joseph Chang, Lisha Chen, John Hartigan (Emeritus), Balaji Raman, Jing Zhang

Mathematics has many aspects: it is the language and tool of the sciences, a cultural phenomenon with a rich historical tradition, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics major provides a broad education in various areas of mathematics in a program flexible enough to accommodate many ranges of interest.

Placement in courses The department offers a three-term sequence in calculus, MATH 112, 115, and 120. Students who have not taken calculus at Yale and who wish to enroll in calculus must take the mathematics online placement examination. They must then bring their exam results, as well as other pertinent information such as Advanced Placement test scores, to the calculus preregistration session, held at the beginning of each term in 432 DL. Advisers will be on hand at the registration session to assist each student in enrolling in the appropriate course. A link to the online placement examination and additional information is available on the departmental Web site (http://math.yale.edu/undergrad/placement-exam).

MATH 112 is an introductory course that presupposes basic skills in high school algebra, geometry, and trigonometry. Enrolling students are expected to know the basic definitions of the trigonometric functions, synthetic division, factorization, and elementary area and volume formulas of plane and solid geometry. MATH 115 presupposes familiarity with the topics covered in MATH 112. MATH 120 presupposes familiarity with the topics covered in MATH 115.

MATH 230, 231 is an advanced course in linear algebra and introductory analysis for students with exceptionally strong backgrounds in mathematics. Students who wish to enroll in MATH 230 should consult with the instructor of the course. After MATH 115, students with a strong interest in abstract mathematics should consider taking MATH 230, 231.

B.A. and B.S. degree programs The prerequisite for each program is calculus through the level of MATH 120, or the equivalent.

Each program normally consists of ten term courses in Mathematics numbered 222 or higher, including MATH 480. These ten may include no more than five term courses from other institutions. Each student is expected to take vector calculus and linear algebra: either MATH 230 and 231, or one of MATH 222 or 225 and MATH 250. To acquire both depth and breadth in the field, students are required to take at least two term courses in each of three of the following five categories: analysis, algebra and number theory, statistics and applied mathematics, geometry and topology, and logic and foundations. Each major program must also include at least one course in at least two of the three core areas: real analysis, algebra, and complex analysis. Taking courses from all three core areas is strongly recommended. To be eligible for Distinction in the Major, a student must have completed at least one course from each of the three core areas. The categories and core areas to which each course belongs are indicated in the course listings.

A candidate for the B.S. degree must take at least two advanced term courses in the physical sciences, such as CHEM 328, 332, 333, or PHYS 401, 402, in addition to the ten term courses required for the B.A. degree. Such courses require the approval of the director of undergraduate studies; written approval is advised.

Any student interested in pursuing further study in pure mathematics should include MATH 301, 305, 310, 350, 370, and 430 in his or her program, and should consider taking one or more graduate-level courses. Students interested in applications of mathematics should include MATH 300 or 301, 310, 350, and a selection of courses from MATH 241, 242, 244, 246, 251, 260, and CPSC 440.

Senior requirement During the senior year students majoring in Mathematics normally take the senior seminar (MATH 480). Alternatively, with the consent of the director of undergraduate studies, highly qualified students may write a senior essay in MATH 470 under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the director of undergraduate studies early in the fall term.
Credit/D/Fail Courses taken Credit/D/Fail may not be counted toward the requirements of the major.

Courses related to mathematics Each Mathematics major is urged to acquire additional familiarity with the uses of mathematics by taking courses in Applied Mathematics, Computer Science, Engineering and Applied Science, Economics, Operations Research, Philosophy, Physics, Statistics, or other departments. In some instances a limited number of such courses may be counted among the ten courses required for the major in Mathematics, with the approval of the director of undergraduate studies.

Each year the Mathematics and Statistics departments offer a large number of graduate courses, some of which are accessible to undergraduates with advanced preparation in mathematics. Further information may be obtained from the directors of undergraduate studies whose permission, with that of the relevant director of graduate studies, is required for admission.

The intensive major Candidates for a degree with an intensive major in Mathematics must take courses in all three of the core areas: real analysis, algebra, and complex analysis. Intensive majors are also expected to include at least two graduate term courses in the Mathematics department, or equivalent independent study, in their programs. Familiarity with the material of the following courses is prerequisite to graduate courses in each category: algebra: two courses between 350 and 399; analysis: MATH 301, 305, 310; algebraic topology: MATH 301, 350; logic and foundations: MATH 270.

Combined B.S./M.S. degree program Students who, by the end of their senior year, complete the requirements of the department for the M.S. in Mathematics will be eligible to receive this degree at their Senior Commencement. Required are: (1) eight term courses numbered 500 or higher, most of which must be completed with grades of B or better; (2) a reading knowledge of mathematical literature in a foreign language of importance for mathematical research (normally French, German, or Russian); (3) satisfactory performance on a general oral examination.

The master’s program is in no sense a substitute for the B.A. or B.S. program; rather, it is designed to accommodate a very few exceptional students who, by means of accelerated or independent study, can satisfy the department as to their command of the content of the normal undergraduate program. Candidates must submit a proposal that foresees this level of achievement by the end of junior year to the director of undergraduate studies no later than the last day of classes in their fifth term of enrollment in Yale College. If approved by the department this proposal will be forwarded to the Dean’s Office. Students’ status and progress will be reviewed before they are permitted to continue in the program in the senior year. For more information on Yale College requirements for the program, see "Simultaneous Award of the Bachelor’s and Master’s Degrees" under "Special Arrangements" in the Academic Regulations.

Students take at least two graduate term courses in the junior year (normally courses in algebra or analysis are the first graduate courses taken). The general oral examination covers a list of topics available from the director of graduate studies and will be accepted in lieu of the usual senior oral presentation. Details concerning the requirements for the master's degree may be obtained from the director of graduate studies.

REQUIREMENTS OF THE MAJOR

Prerequisite MATH 120 or equivalent

Number of courses B.A. - 10 term courses numbered 222 or higher, incl MATH 480; B.S. - same, with 2 addtl courses in physical sciences

Specific courses required MATH 230 and 231; or MATH 222 or 225, and MATH 250

Distribution of courses B.A. - 2 courses in each of 3 categories chosen from analysis, algebra and number theory, stat and applied math, geometry and topology, logic and foundations; courses from 2 of 3 core areas, as specified; B.S. - same, with 2 addtl advanced courses in physical sciences approved by DUS

Substitution permitted With DUS permission, certain courses in Applied Math, Comp Sci, Engineering & Applied Science, Econ, Operations Research, Phil, Physics, Stat

Intensive major Courses in all 3 core areas; 2 grad courses or equivalent independent study counted among the required courses

Senior requirement Senior sem (MATH 480) or, with DUS permission, senior essay (MATH 470) and oral report

Courses

*MATH 101b, Geometry of Nature Michael Frame
Geometric patterns in nature, including classical models of spirals in seashells and sunflowers, symmetry of honeycombs and snowflakes, and the curvature of soap films; the shape of the universe; ways to visualize the fourth dimension; and a brief introduction to fractal geometry. Enrollment limited to freshmen and sophomores who have not previously taken a high school or college calculus course. QR

*MATH 107a, Mathematics in the Real World Anna Lachowska
The use of mathematics to address real-world problems. Applications of exponential functions to compound interest and population growth; geometric series in mortgage payments, amortization of loans, present value of money, and drug doses and blood levels; basic probability, Bayes’s rule, and false positives in drug testing; elements of logic. No knowledge of calculus required. Enrollment limited to students who have not previously taken a high school or college calculus course. QR
MATH 108, Estimation and Error
MATH 109, History of Mathematics

*MATH 112a or b, Calculus of Functions of One Variable I  Andrew Casson and staff
Limits and their properties. Definitions and some techniques of differentiation and the evaluation of definite integrals, with applications. Use of the software package Mathematica to illustrate concepts. No prior acquaintance with calculus or computing assumed. Online preregistration is required; see Calculus Advising and Preregistration (http://math.yale.edu/calculus-advising-and-preregistration) for details.  QR

*MATH 112a or b, Calculus of Functions of One Variable II  James Rolf
A continuation of MATH 112. Applications of integration, with some formal techniques and numerical methods. Improper integrals, approximation of functions by polynomials, infinite series. Exercises involve the software package Mathematica. After MATH 112 or equivalent; open to freshmen with some preparation in calculus. May not be taken after MATH 116. Online preregistration is required; see Calculus Advising and Preregistration (http://math.yale.edu/calculus-advising-and-preregistration) for details.  QR

*MATH 116a, Mathematical Models in the Biosciences I: Calculus Techniques  Michael Frame
Introduction to topics in mathematical modeling that are applicable to biological systems. Discrete and continuous models of population, neural, and cardiac dynamics. Stability of fixed points and limit cycles of differential equations. Applications include Norton’s chemotherapy scheduling and stochastic models of tumor suppressor gene networks. After MATH 112 or equivalent. May not be taken after MATH 115.  QR

*MATH 118a or b, Introduction to Functions of Several Variables  Marketa Havlickova [F] and James Rolf [Sp]
A combination of linear algebra and differential calculus of several variables. Matrix representation of linear equations, Gauss elimination, vector spaces, independence, basis and dimension, projections, least squares approximation, and orthogonality. Three-dimensional geometry, functions of two and three variables, level curves and surfaces, partial derivatives, maxima and minima, and optimization. Intended for students in the social sciences, especially Economics. May not be taken after MATH 120 or 222. Prerequisite: MATH 112.  QR

*MATH 120a or b, Calculus of Functions of Several Variables  Marketa Havlickova
Analytic geometry in three dimensions, using vectors. Real-valued functions of two and three variables, partial derivatives, gradient and directional derivatives, level curves and surfaces, maxima and minima. Parametrized curves in space, motion in space, line integrals; applications. Multiple integrals, with applications. Divergence and curl. The theorems of Green, Stokes, and Gauss. After MATH 115, or with permission of instructor. May not be taken after MATH 121. Online preregistration is required; see Calculus Advising and Preregistration (http://math.yale.edu/calculus-advising-and-preregistration) for details.  QR

*MATH 121a or b, Calculus of Functions of One Variable II  Andrew Casson and staff
Limits and their properties. Definitions and some techniques of differentiation and the evaluation of definite integrals, with applications. Use of the software package Mathematica to illustrate concepts. No prior acquaintance with calculus or computing assumed. Online preregistration is required; see Calculus Advising and Preregistration (http://math.yale.edu/calculus-advising-and-preregistration) for details.  QR

*MATH 122a or b, Linear Algebra with Applications  Amanda Folsom
Math: Algebra/Number Theory

MATH 225a or b, Linear Algebra and Matrix Theory  Van Vu [F] and Asher Auel [Sp]
An introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, and quadratic forms. Some relations to calculus and geometry are included. After or concurrently with MATH 120. May not be taken after MATH 222.  QR
Math: Algebra/Number Theory
Psychology: AdvSci NeuroTrk

[ MATH 228, From Euclid to Einstein ]

*MATH 230a, Vector Calculus and Linear Algebra I  Asaf Hadari
A careful study of the calculus of functions of several variables, combined with linear algebra.  QR
Math: Algebra/Number Theory

*MATH 231b, Vector Calculus and Linear Algebra II  Asaf Hadari
Continuation of MATH 230. Application of linear algebra to differential calculus. Inverse and implicit function theorems; the idea of a
manifold; integration of differential forms; general Stokes’ theorem.  QR
Math: Analysis

*MATH 235b, Reflection Groups  Anna Lachowska
Concepts of linear algebra are used to explore the algebraic and geometric properties of groups generated by reflections. Examples from
reflection groups introduce elements of group theory, Lie algebras, and representation theory. Reflections in a real Euclidean space,
groups generated by reflections, crystallographic groups, and Coxeter groups. Preference to sophomores majoring in mathematics or the
sciences. Prerequisite: MATH 222 or 225.  QR
Math: Algebra/Number Theory

MATH 241a / STAT 241aG, Probability Theory  Huibin Zhou
Introduction to probability theory. Topics include probability spaces, random variables, expectations and probabilities, conditional
probability, independence, discrete and continuous distributions, central limit theorem, Markov chains, and probabilistic modeling. After
or concurrently with MATH 120 or equivalent.  QR
Math: Stat/Applied Math

MATH 242b / STAT 242bG, Theory of Statistics  Lisha Chen
Study of the principles of statistical analysis. Topics include maximum likelihood, sampling distributions, estimation, confidence
intervals, tests of significance, regression, analysis of variance, and the method of least squares. Some statistical computing. After STAT
241 and concurrently with or after MATH 222 or 225, or equivalents.  QR
Math: Stat/Applied Math

MATH 244a / AMTH 244a, Discrete Mathematics  Staff
Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients,
Stirling numbers. Properties of finite set systems. Recommended preparation: MATH 115 or equivalent.  QR
Math: Stat/Applied Math

MATH 246a or b, Ordinary Differential Equations  Andrei Osipov [F] and Kim Dang [Sp]
First-order equations, second-order equations, linear systems with constant coefficients. Numerical solution methods. Geometric and
algebraic properties of differential equations. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or equivalent.  QR
Math: Analysis

MATH 247b / AMTH 247b / G&G 247b, Partial Differential Equations  Igor Frenkel
Introduction to partial differential equations, wave equation, Laplace’s equation, heat equation, method of characteristics, calculus of
variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225, MATH 246, and ENAS 194, or
equivalents.  QR
Math: Stat/Applied Math

MATH 250a, Vector Analysis  Roger Howe
Calculus of functions of several variables, using vector and matrix methods. The derivative as a linear mapping. Inverse and implicit
forms. Theorems of Green and Gauss; general Stokes’ theorem. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or equivalent.  QR
Math: Analysis

MATH 251b / STAT 251b, Stochastic Processes  Staff
Introduction to the study of random processes, including Markov chains, Markov random fields, martingales, random walks, Brownian
motion, and diffusions. Techniques in probability, such as coupling and large deviations. Applications chosen from image reconstruction,
Bayesian statistics, finance, probabilistic analysis of algorithms, and genetics and evolution. After STAT 241 or equivalent.  QR
Math: Stat/Applied Math

MATH 260a, Vector Analysis  Roger Howe
Calculus of functions of several variables, using vector and matrix methods. The derivative as a linear mapping. Inverse and implicit
forms. Theorems of Green and Gauss; general Stokes’ theorem. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or equivalent.  QR
Math: Analysis

MATH 290b, Fractal Geometry: Concepts and Applications  Amanda Folsom
An examination of mathematical patterns repeating on many levels. Mathematical concepts of fractals and chaos, and application of these
tools to modeling natural phenomena. Prerequisites: MATH 120 and 222 or equivalent.  QR
Math: Geometry/Topology

MATH 300b, Topics in Analysis  Daniel Fresen
An introduction to analysis, with topics chosen from infinite series, the theory of metric spaces, and fixed-point theorems with applications. Students who have taken MATH 230, 231 should take MATH 301 instead of this course. After MATH 250 or with permission of instructor. QR
Math: Analysis
Math: Core Real Analysis

* MATH 301a, Introduction to Analysis  Peter Jones
Foundations of real analysis, including metric spaces and point set topology, infinite series, and function spaces. After MATH 230, 231 or equivalent. QR
Math: Analysis
Math: Core Real Analysis

MATH 305bG, Real Analysis  Yair Minsky
The Lebesgue integral, Fourier series, applications to differential equations. After MATH 301 or with permission of instructor. QR
Math: Analysis
Math: Core Real Analysis

MATH 310a, Introduction to Complex Analysis  Gregory Margulis
Math: Core Complex Analysis
Math: Analysis

* MATH 315bG, Intermediate Complex Analysis  Mikhail Kapranov
Continuation of MATH 310. Topics may include argument principle, Rouché's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem. Functions of finite order, Hadamard's theorem, meromorphic functions. Mittag-Leffler's theorem, subharmonic functions. After MATH 310. QR RP
Math: Core Complex Analysis
Math: Analysis

* MATH 320aG, Measure Theory and Integration  Howard Garland
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals. After MATH 305 or equivalent. QR RP
Math: Analysis
Math: Core Real Analysis

* MATH 325bG, Introduction to Functional Analysis  Gregory Margulis
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 320. QR RP
Math: Analysis
Math: Core Real Analysis

MATH 330b / STAT 330bG, Advanced Probability  David Pollard
Measure theoretic probability, conditioning, laws of large numbers, convergence in distribution, characteristic functions, central limit theorems, martingales. Some knowledge of real analysis assumed. QR
Math: Stat/Applied Math

* MATH 345b, Modern Combinatorics  Van Vu
Recent developments and important questions in combinatorics. Relations to other areas of mathematics such as analysis, probability, and number theory. Topics include probabilistic method, random graphs, random matrices, pseudorandomness in graph theory and number theory, Szemeredi's theorem and lemma, and Green-Tao's theorem. Prerequisite: MATH 244. QR
Math: Stat/Applied Math

MATH 350a, Introduction to Abstract Algebra  Andrew Casson
Group theory, structure of Abelian groups, and applications to number theory. Symmetric groups and linear groups including orthogonal and unitary groups; properties of Euclidean and Hermitian spaces. Some examples of group representations. Modules over Euclidean rings, Jordan and rational canonical forms of a linear transformation. After MATH 222 or equivalent. QR
Math: Core Algebra
Math: Algebra/Number Theory
### [ MATH 353, Introduction to Representation Theory ]

**MATH 360a, Introduction to Lie Groups**  
Roger Howe  
Lie groups as the embodiment of the idea of continuous symmetry. The exponential map on matrices and applications; spectral theory; examples and structure of Lie groups and Lie algebras; connections with geometry and physics. After MATH 231 or 250 or equivalent. MATH 300 or 301 and MATH 350 recommended.  
Math: Algebra/Number Theory

**MATH 370b, Fields and Galois Theory**  
Marketa Havlickova  
Rings, with emphasis on integral domains and polynomial rings. The theory of fields and Galois theory, including finite fields, solvability of equations by radicals, and the fundamental theorem of algebra. Quadratic forms. After MATH 350.  
Math: Core Algebra

**MATH 373a, Algebraic Number Theory**  
Alexander Goncharov  
Structure of fields of algebraic numbers (solutions of polynomial equations with integer coefficients) and their rings of integers; prime decomposition of ideals and finiteness of the ideal class group; completions and ramification; adeles and ideles; zeta functions.  
Prerequisites: MATH 310 and 370.  
Math: Algebra/Number Theory

**MATH 380a, Modern Algebra I**  
Staff  
A survey of algebraic constructions and theories at a sophisticated level. Topics include categorical language, free groups and other free objects in categories, general theory of rings and modules, artinian rings, and introduction to homological algebra. After MATH 350 and 370.  
Math: Core Algebra

**MATH 381b, Modern Algebra II**  
Gregg Zuckerman  
Topics in commutative algebra: general extension of fields; Noetherian, local, and Dedekind rings. Introduction to valuation theory.  
Math: Core Algebra

**MATH 400b, Introduction to Mathematical Mechanics**  
Vincent Moncrief  
Newton’s equations and the Galilean group; the Euler-Lagrange equations and Noether’s theorem; the Kepler problem and rigid body motion; symplectic manifolds and Hamiltonian mechanics. After MATH 120, and 222 or 225, or equivalents.  
Math: Stat/Applied Math

**MATH 430b, Introduction to Algebraic Topology**  
Staff  
The theory of fundamental groups and covering spaces, with particular reference to two-dimensional manifolds. After MATH 350, and 300 or 301, or equivalents.  
Math: Geometry/Topology

### [ MATH 435, Differential Geometry ]

**MATH 470a or b, Individual Studies**  
Andrew Casson and staff  
Individual investigation of an area of mathematics outside of those covered in regular courses, involving directed reading, discussion, and either papers or an examination. A written plan of study approved by the student’s adviser and the director of undergraduate studies is required. The course may normally be elected for only one term.  
Math: Stat/Applied Math

**MATH 480a or b, Senior Seminar: Mathematical Topics**  
Mikhail Kapranov  
A number of mathematical topics are chosen each term — e.g., differential topology, Lie algebras, mathematical methods in physics — and explored in one section of the seminar. Students present several talks on the chosen topic. One section each year is devoted to topics of interest to Economics and Mathematics majors, and is co-taught by a member of the Economics department.

### Other Courses Related to Mathematics

**CPSC 201a or b, Introduction to Computer Science**  
Dana Angluin  
Introduction to the concepts, techniques, and applications of computer science. Topics include computer systems (the design of computers and their languages); theoretical foundations of computing (computability, complexity, algorithm design); and artificial intelligence (the organization of knowledge and its representation for efficient search). Examples stress the importance of different problem-solving methods. After CPSC 112 or equivalent.  
Math: Stat/Applied Math

**CPSC 365b, Design and Analysis of Algorithms**  
Daniel Spielman  
Paradigms for problem solving: divide and conquer, recursion, greedy algorithms, dynamic programming, randomized and probabilistic algorithms. Techniques for analyzing the efficiency of algorithms and designing efficient algorithms and data structures. Algorithms for
Mathematics

graph theoretic problems, network flows, and numerical linear algebra. Provides algorithmic background essential to further study of computer science. After CPSC 202 and 223. QR
Math: Stat/Applied Math

**CPSC 440b**
**Numerical Computation** Vladimir Rokhlin
Algorithms for numerical problems in the physical, biological, and social sciences: solution of linear and nonlinear systems of equations, interpolation and approximation of functions, numerical differentiation and integration, optimization. After CPSC 112 or an equivalent introductory programming course; MATH 120; and MATH 222 or 225 or CPSC 202. QR
Math: Stat/Applied Math

**OPRS 235a / AMTH 235a, Optimization** Eric Denardo
Resource allocation problems solved by linear programming and its generalizations: the simplex method, duality, the Karush-Kuhn-Tucker conditions for nonlinear programs, economic equilibria, and selected applications. Prerequisite: MATH 118, 222, or 225, or equivalent. QR

**PHIL 267a**
**Mathematical Logic** Sun-Joo Shin
An introduction to the metatheory of first-order logic, up to and including the completeness theorem for the first-order calculus. Introduction to the basic concepts of set theory. Prerequisite: PHIL 115 or permission of instructor. QR
Math: Logic/Foundations

*PHIL 427b*
**Computability and Logic** Sun-Joo Shin
A technical exposition of Gödel's first and second incompleteness theorems and of some of their consequences in proof theory and model theory, such as Löb's theorem, Tarski's undefinability of truth, provability logic, and nonstandard models of arithmetic. Prerequisite: PHIL 267 or permission of instructor. QR, HU
Math: Logic/Foundations