MATHEMATICS

10 Hillhouse Avenue, 203.432.7058
http://math.yale.edu
M.S., M.Phil., Ph.D.

Chair
Yair Minsky

Director of Graduate Studies
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Professors Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Gil Kalai (Adjunct), Ivan Losev, Alexander Lubotzky (Adjunct), Gregory Margulis, Yair Minsky, Vincent Moncريف (Physics), Hee Oh, Sam Payne, Nicholas Read (Physics; Applied Physics), Vladimir Rokhlin (Computer Science), Wilhelm Schlag, George Seligman (Emeritus), Daniel Spielman (Computer Science), Van Vu, John Wettlaufer (Geology & Geophysics; Physics), Gregg Zuckerman

Associate Professor Yifeng Liu

Assistant Professor Stefan Steinerberger

FIELDS OF STUDY
Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; geometric analysis; kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; combinatorics and discrete mathematics.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE
All students are required to: (1) complete eight term courses at the graduate level, at least two with Honors grades; (2) pass qualifying examinations on their general mathematical knowledge; (3) submit a dissertation prospectus; (4) participate in the instruction of undergraduates; (5) be in residence for at least three years; and (6) complete a dissertation that clearly advances understanding of the subject it considers. The normal time for completion of the Ph.D. program is five years. Requirement (1) should be completed by the end of the second year. A sequence of three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term, at intervals of about one month. All qualifying examinations must be taken by the end of the third term. The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1)–(5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 991, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

HONORS REQUIREMENT
Students must meet the Graduate School’s Honors requirement by the end of the fourth term of full-time study.

TEACHING
Teaching experience is integral to graduate education at Yale. Therefore, most Mathematics students are required to assist in teaching during five terms. Students in years one and two serve as tutors and graders in undergraduate mathematics courses during one term per year. The department also offers a required teaching practicum in year two. In years three through five, students normally teach one section of calculus or its equivalent during one term per year. Students receiving external fellowships may petition for a waiver of teaching while receiving external funding in place of University funding, but they are still required to teach one section of calculus or its equivalent for a minimum of two terms over the course of their program.

MASTER’S DEGREES
M.Phil. In addition to the Graduate School’s Degree Requirements (see under Policies and Regulations), a student must undertake a reading program of at least two terms’ duration in a specific significant area of mathematics under the supervision of a faculty adviser and demonstrate command of the material studied during the reading period at a level sufficient for teaching and research.

M.S. (en route to the Ph.D.) A student must complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year. The M.S. degree is conferred only en route to the Ph.D.; there is no separate master’s program in Mathematics.
COURSES

MATH 500a, Modern Algebra I  Yifeng Liu
A survey of algebraic constructions and theories at a sophisticated level. Topics include categorical language, free groups and other free objects in categories, general theory of rings and modules, artinian rings, and introduction to homological algebra.

MATH 520a, Measure Theory and Integration  Arie Levit
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 544a, Introduction to Algebraic Topology I  Yair Minsky
A one-term graduate introductory course in algebraic topology. We discuss algebraic and combinatorial tools used by topologists to encode information about topological spaces. Broadly speaking, we study the fundamental group of a space, its homology, and its cohomology. While focusing on the basic properties of these invariants, methods of computation, and many examples, we also see applications toward proving classical results. These include the Brouwer fixed-point theorem, the Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher’s Algebraic Topology, which is available for free on his website.

MATH 573b, Algebraic Number Theory  Asher Auel
Structure of fields of algebraic numbers (solutions of polynomial equations with integer coefficients) and their rings of integers; prime decomposition of ideals and finiteness of the ideal class group; completions and ramification; adeles and ideles; zeta functions.

MATH 600a, Topics in Homogeneous Dynamics and Geometry  Hee Oh
We discuss a variety of topics in homogeneous dynamics, especially in relation to hyperbolic manifolds of infinite volume. Possible topics include: (1) basics in hyperbolic geometry; (2) constructions of (non-) arithmetic lattices and rigid acylindrical hyperbolic manifolds; (3) mixing of the frame flow, and applications to counting and equidistribution problems; (4) the spectrum of the Laplacian, expanders, and effective mixing; (5) orbit closures on the frame bundle of a hyperbolic 3-manifold; (6) Margulis function and its applications; (7) linearization and limit distribution of orbits of unipotent flows; (8) orbit closures on the frame bundle of a hyperbolic n-manifold.

MATH 608a, Introduction to Arithmetic Geometry  Asher Auel
This course explores some of the major themes in arithmetic geometry. Topics include Galois cohomology and descent, principal homogeneous spaces, quadratic forms and the Brauer group, Milnor K-theory, as well as the existence of rational points. Of particular interest are varieties over finite fields, number fields, and function fields. Prerequisites: experience with algebra and Galois theory is necessary. Some exposure to algebraic geometry is useful, but the course may be taken in conjunction with a beginning algebraic geometry course.

MATH 666a, Classical Statistical Thermodynamics  John Wettlaufer
Classical thermodynamics is derived from statistical thermodynamics. Using the multi-particle nature of physical systems, we derive ergodicity, the central limit theorem, and the elemental description of the second law of thermodynamics. We then develop kinetics, transport theory, and reciprocity from the linear thermodynamics of irreversible processes. Topics of focus include Onsager reciprocal relations, the Fokker-Planck equation, stability in the sense of Lyapunov, and time invariance symmetry. We explore phenomena that are of direct relevance to astrophysical and geophysical settings. No quantum mechanics is necessary as a prerequisite.

MATH 670a, Topics on Random Graphs  Mathias Schacht
We discuss a variety of topics on the theory of random graphs. We introduce the standard models of random graphs and focus on the threshold phenomenon for graph properties. For many interesting and natural graph properties, the probability for a random graph to enjoy the property moves from close to 0 to close to 1 in a relatively small interval in terms of the given density of the random graph; we investigate this for the properties of (1) containing fixed size or spanning subgraphs (like a perfect matching or a Hamiltonian cycle); (2) chromatic number; (3) transfer of classical theorems in extremal combinatorics. This course is open to students from Statistics and Computer Science as well. Yale College juniors and seniors are also welcome. Some background in discrete probability and graph theory is helpful, but the course is self-contained.

MATH 674b, Extremal Combinatorics  Patrick Devlin
The course is a standalone introduction to extremal combinatorics. We focus on algebraic and combinatorial techniques (e.g., combinatorial Nullstellensatz, tensor constructions, duality, shifting) as applied to a variety of discrete settings including hypergraphs and set systems, designs, graphs, posets, and arithmetic combinatorics. Time permitting, we may also discuss topics from eigenvalue methods, entropy, linear programming, finite geometry, or coding theory. No prerequisites. Interested undergraduates are encouraged to contact the instructor.

MATH 675a, Numerical Methods for Partial Differential Equations  Vladimir Rokhlin
function of the intersection numbers should satisfy the differential equations of the KdV hierarchy. For this, we provide a quick overview

This part culminates with discussion of Witten's conjecture on a two-dimensional quantum gravity model that a certain generating

Mumford compactifications, consider some of their natural cohomology classes, and study the intersection numbers between them.

complex analysis, geometric topology, algebraic geometry, arithmetic geometry, and mathematical physics. We introduce their Deligne-

hierarchies, graphs on surfaces, complex geometry, and singularity theory, using certain ideas from theoretical physics as a main

In this course, we discuss the interconnection between several important subjects in mathematics, including moduli theory, integrable

MATH 802a, Mathematics of Two-Dimensional Quantum Gravity and Topological Strings

In this course, we discuss the interconnection between several important subjects in mathematics, including moduli theory, integrable

hierarchies, graphs on surfaces, complex geometry, and singularity theory, using certain ideas from theoretical physics as a main

This part culminates with discussion of Witten's conjecture on a two-dimensional quantum gravity model that a certain generating

function of the intersection numbers should satisfy the differential equations of the KdV hierarchy. For this, we provide a quick overview
of integrable hierarchies and then sketch Kontsevich’s proof of the conjecture. In the second part of the course, we discuss mathematics of topological string theory, especially of what is usually referred to as the B-model in the subject. Mathematically speaking, this amounts to the study of complex algebraic geometry and Hodge theory of Calabi-Yau manifolds on the one hand and singularity theory on the other. Our aim is to understand both subjects in a uniform and coherent way, as motivated by ideas of topological string theory. Our treatment is built on the rigorous mathematical frameworks developed by several people in and around the subject of mirror symmetry, including Barannikov, Dubrovin, Givental, Kontsevich, and K. Saito. Class material is entirely mathematical and in particular does not assume any background on physics; physics will be used only as a guiding principle for the choice of topics and a presentation. On the other hand, the course assumes basic knowledge of complex analysis, algebraic topology, differential geometry, and algebraic geometry; references are provided for those who are less prepared but willing to spend extra time to follow the course.

MATH 827b, Lang Teaching Seminar  Marketa Havlickova
This course prepares graduate students for teaching calculus classes. It is a mix of theory and practice, with topics such as preparing classes, presenting new concepts, choosing examples, encouraging student participation, grading fairly and effectively, implementing active learning strategies, and giving and receiving feedback. Open only to mathematics graduate students in their second year.

MATH 845a, Introduction to Algebraic Geometry  Kalina Mincheva

MATH 991a / CPSC 991a, Ethical Conduct of Research  Alexander Goncharov