APPLIED MATHEMATICS

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http://applied.math.yale.edu
M.S., M.Phil., Ph.D.

Director of Graduate Studies
Vladimir Rokhlin

Professors Andrew Barron (*Statistics & Data Science*), Joseph Chang (*Statistics & Data Science*), Ronald Coifman (*Mathematics; Computer Science*), John Emerson (*Adjunct; Statistics & Data Science*), Thierry Emonet (*Molecular, Cellular, & Developmental Biology; Physics*), Michael Fischer (*Computer Science*), Anna Gilbert (*Mathematics; Statistics & Data Science*), Jonathon Howard (*Molecular Biophysics & Biochemistry*), Peter Jones (*Mathematics*), Yuval Kluger (*Pathology*), Nicholas Read (*Physics; Applied Physics; Mathematics*), Vladimir Rokhlin (*Computer Science; Mathematics*), Wilhelm Schlag (*Mathematics*), John Schotland (*Mathematics*), Mitchell Smooke (*Mechanical Engineering & Materials Science; Applied Physics*), Daniel Spielman (*Computer Science; Mathematics*), Van Vu (*Mathematics*), John Wettlaufer (*Earth & Planetary Sciences; Mathematics; Physics*), Huibin Zhou (*Statistics & Data Science*), Steven Zucker (*Computer Science; Biomedical Engineering*)

Associate Professors Josephine Hoh (*Public Health*), Smita Krishnaswamy (*Genetics; Computer Science*), Sekhar Tatikonda (*Statistics & Data Science*)

Assistant Professor Roy Lederman (*Statistics & Data Science*)

FIELDS OF STUDY

The graduate Program in Applied Mathematics comprises the study and application of mathematics to problems motivated by a wide range of application domains. Areas of concentration include the analysis of data in very high-dimensional spaces, the geometry of information, computational biology, and randomized algorithms. Topics covered by the program include classical and modern applied harmonic analysis, linear and nonlinear partial differential equations, numerical analysis, scientific computing and applications, discrete algorithms, combinatorics and combinatorial optimization, graph algorithms, geometric algorithms, discrete mathematics and applications, cryptography, statistical theory and applications, probability theory and applications, information theory, econometrics, financial mathematics, statistical computing, and applications of mathematical and computational techniques to fluid mechanics, combustion, and other scientific and engineering problems.

INTEGRATED GRADUATE PROGRAM IN PHYSICAL AND ENGINEERING BIOLOGY (PEB)

Students applying to the Ph.D. program in Applied Mathematics may also apply to be part of the PEB program. See the description under Non-Degree-Granting Programs, Councils, and Research Institutes for course requirements, and http://peb.yale.edu for more information about the benefits of this program and application instructions.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE

All students are required to: (1) complete twelve term courses (including reading courses) at the graduate level, at least two with Honors grades; (2) pass a qualifying examination on their general applied mathematical knowledge (in algebra, analysis, and probability and statistics) by the end of their second year; (3) submit a dissertation prospectus; (4) participate in the instruction of undergraduates; (5) be in residence for at least three years; and (6) complete a dissertation that clearly advances understanding of the subject it considers. Prior to registering for a second year of study, and in addition to all other academic requirements, students must successfully complete MATH 991, Ethical Conduct of Research, or another approved course on responsible conduct in research. Teaching is considered an integral part of training at Yale University, so all students are expected to complete two terms of teaching within their first two years. Students who require additional support from the Graduate School will be required to teach additional terms, if needed, after they have fulfilled the academic teaching requirement.

Requirement (1) normally includes four core courses in each of the methods of applied analysis, numerical computation, algorithms, and probability; these should be taken during the first year. The qualifying examination is normally taken by the end of the third term and will test knowledge of the core courses as well as more specialized topics. The thesis is expected to be independent work, done under the guidance of an adviser. An adviser is usually contacted not long after the student passes the qualifying examinations; students are encouraged to find an adviser sooner rather than later. A student is admitted to candidacy after completing requirements (1)–(5) and finding an adviser.

In addition to the above, all first-year students must successfully complete one course on the responsible conduct of research (e.g., MATH 991 or CPSC 991) and AMTH 525, Seminar in Applied Mathematics.

HONORS REQUIREMENT

Students must meet the Graduate School's Honors requirement by the end of the fourth term of full-time study.

M.D./Ph.D. Students

With permission of the DGS, M.D./Ph.D. students may request a reduction in the program's academic teaching requirement to one term of teaching. Only students who teach are eligible to receive a University stipend contingent on teaching.

Master's Degrees

M.Phil. The minimum requirements for this degree are that a student shall have completed all requirements for the Applied Math Ph.D. program as described above except the required teaching, the prospectus, and the dissertation. Students will not generally have satisfied the requirements for the M.Phil. until after two years of study, except where graduate work done before admission to Yale has reduced the student's graduate course work at Yale. In no case will the degree be awarded after less than one year of residence in the Yale Graduate School of Arts and Sciences. See also Degree Requirements under Policies and Regulations.

M.S. (en route to the Ph.D.) Applications for a terminal master's degree are not accepted. Students who withdraw from the Ph.D. program may be eligible for the M.S. degree if they have completed ten graduate-level term courses, maintained a High Pass average, and met the Graduate School's Honors requirement for the Ph.D. program. Students who are eligible for or who have already received the M.Phil. will not be awarded the M.S.

More information is available on the program's website, http://applied.math.yale.edu.

Courses

AMTH 525a or b, Seminar in Applied Mathematics Boris Landa
This course consists of weekly seminar talks given by a wide range of speakers. Required of all first-year students.

AMTH 552b / CB&B 663b / CPSC 552b, Deep Learning Theory and Applications Smita Krishnaswamy
Deep neural networks have gained immense popularity within the past decade due to their success in many important machine-learning tasks such as image recognition, speech recognition, and natural language processing. This course provides a principled and hands-on approach to deep learning with neural networks. Students master the principles and practices underlying neural networks, including modern methods of deep learning, and apply deep learning methods to real-world problems including image recognition, natural language processing, and biomedical applications. Course work includes homework, a final exam, and a final project—either group or individual, depending on enrollment—with both a written and oral (i.e., presentation) component. The course assumes basic prior knowledge in linear algebra and probability. Prerequisites: CPSC 202 and knowledge of Python programming.

AMTH 553a / CB&B 555a / CPSC 553a / GENE 555a, Unsupervised Learning for Big Data Smita Krishnaswamy
This course focuses on machine-learning methods well-suited to tackling problems associated with analyzing high-dimensional, high-throughput noisy data including: manifold learning, graph signal processing, nonlinear dimensionality reduction, clustering, and information theory. Though the class goes over some biomedical applications, such methods can be applied in any field. Prerequisites: knowledge of linear algebra and Python programming.

AMTH 617b / MATH 617b, Partial Differential Equations John Schotland
Classical theory of Laplace, heat and wave equations including energy methods and maximum principles; distribution theory and the Fourier transform; Sobolev spaces; elliptic boundary value problems. The latter part of the course emphasizes functional analytic techniques and estimates rather than explicit solutions.

AMTH 631a / S&DS 631a, Optimization and Computation Anna Gilbert
An introduction to optimization and computation motivated by the needs of computational statistics, data analysis, and machine learning. This course provides foundations essential for research at the intersections of these areas, including the asymptotic analysis of algorithms, an understanding of condition numbers, conditions for optimality, convex optimization, gradient descent, linear and conic programming, and NP hardness. Model problems come from numerical linear algebra and constrained least squares problems. Other useful topics include data structures used to represent graphs and matrices, hashing, automatic differentiation, and randomized algorithms. Prerequisites: multivariate calculus, linear algebra, probability, and permission of the instructor. Enrollment is limited, with preference given to graduate students in Statistics and Data Science.

AMTH 640b / CPSC 640b, Topics in Numerical Computation Staff
This course discusses several areas of numerical computing that often cause difficulties to non-numericists, from the ever-present issue of condition numbers and ill-posedness to the algorithms of numerical linear algebra to the reliability of numerical software. The course also provides a brief introduction to "fast" algorithms and their interactions with modern hardware environments. The course is addressed to Computer Science graduate students who do not necessarily specialize in numerical computation; it assumes the understanding of calculus and linear algebra and familiarity with (or willingness to learn) either C or FORTRAN. Its purpose is to prepare students for using elementary numerical techniques when and if the need arises.

AMTH 675a / MATH 675a, Numerical Methods for Partial Differential Equations Vladimir Rokhlin
(1) Review of the classical qualitative theory of ODEs; (2) Cauchy problem. Elementary numerical methods: Euler, Runge-Kutta, predictor-corrector. Stiff systems of ODEs: definition and associated difficulties, implicit Euler, Crank-Nicolson, barrier theorems. Richardson extrapolation and deferred corrections; (3) Boundary value problems. Elementary theory: finite differences, finite elements, abstract formulation and related spaces, integral formulations and associated numerical tools, nonlinear problems; (4) Partial differential equations (PDEs). Introduction: counterexamples, Cauchy–Kowalevski theorem, classification of second-order PDEs, separation of

variables; (5) Numerical methods for elliptic PDEs. Finite differences, finite elements, Richardson and deferred corrections, Lippmann–Schwinger equation and associated numerical tools, classical potential theory, “fast” algorithms; (6) Numerical methods for parabolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, integral formulations and related numerical tools; (7) Numerical methods for hyperbolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, time-invariant problems and Fourier transform.

AMTH 701b / MATH 701b, Topics in Analysis Peter Jones
This course provides an introduction to some topics in harmonic analysis and probability. Starting with basic dyadic analysis, we use this to give a short introduction to stochastic processes. We then give an introduction to quasiconformal mappings and results concerning random Jordan curves in R2. The main theorem discussed at the end of the course is contained in K. Astala, P. Jones, A. Kupiainen, E. Saksman, “Random Conformal Weldings,” *Acta Mathematica* 207 (2011): 203–254. Some of the topics to be covered are: dyadic grids, maximal functions, and domain decomposition; Haar wavelet analysis, square functions, and Lp estimates; positive measures and product formulas, dyadic earth mover distances; wavelets and applications to function spaces; probability theory in the dyadic setting and the martingale convergence theorem; random walk, Brownian motion (via Haar functions) and introduction to stochastic processes, Feynman-Kac formalism; Brownian motion and relations to L2. Other topics covered depend on students’ interests and could include: the Johnson-Lindenstrauss theorem and relations to random Gaussian vectors; the Gaussian Free Field and Kahane’s theorem on exponentiation of the GFF; multiscale estimates for Kahane’s theorem; degenerate QC mappings and applications related to Kahane’s theorem on the GFF. A background in basic graduate-level analysis (e.g., MATH 320 and MATH 325) is assumed, though most of the material can be understood by anyone with an understanding of Lebesgue measure.

AMTH 710a / MATH 710a, Harmonic Analysis on Graphs and Applications Ronald Coifman
This class covers basic methods of classical harmonic analysis that can be carried over to graphs and data analysis. We cover the fundamentals of nonlinear Fourier analysis, including functional approximations in high dimensions. We intend to cover foundational material useful for data organization and geometries.

AMTH 765b / CB&B 562b / ENAS 561b / INP 562b / MB&B 562b / MCDB 562b / PHYS 562b, Modeling Biological Systems II Thierry Emonet, Joe Howard, and Damon Clark
This course covers advanced topics in computational biology. How do cells compute, how do they count and tell time, how do they oscillate and generate spatial patterns? Topics include time-dependent dynamics in regulatory, signal-transduction, and neuronal networks; fluctuations, growth, and form; mechanics of cell shape and motion; spatially heterogeneous processes; diffusion. This year, the course spends roughly half its time on mechanical systems at the cellular and tissue level, and half on models of neurons and neural systems in computational neuroscience. Prerequisite: a 200-level biology course or permission of the instructor.

AMTH 999b, Directed Reading Vladimir Rokhlin