MATHEMATICS

10 Hillhouse Avenue, 203.432.7058
http://math.yale.edu
M.S., M.Phil., Ph.D.

Chair
Wilhelm Schlag

Director of Graduate Studies
Ivan Loseu

Professors Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Anna Gilbert, Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Richard Kenyon, Ivan Loseu, Alexander Lubotzky (Adjunct), Gregory Margulis (Emeritus), Yair Minsky, Vincent Moncrief (Physics), Andrew Neitzke, Hee Oh, Nicholas Read (Physics; Applied Physics), Vladimir Rokhlin (Computer Science), Wilhelm Schlag, John Schotland, George Seligman (Emeritus), Charles Smart, Daniel Spielman (Computer Science), Van Vu, Lu Wang, John Wettlaufer (Earth & Planetary Sciences; Physics), Gregg Zuckerman (Emeritus)

Assistant Professor Junliang Shen

FIELDS OF STUDY
Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; probability; Kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; combinatorics and discrete mathematics.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE
In order to qualify for the Mathematics Ph.D., all students are required to:

1. Complete eight term courses at the graduate level, at least two with Honors grades.
2. Pass qualifying examinations on their general mathematical knowledge;
3. Submit a dissertation prospectus;
4. Participate in the instruction of undergraduates;
5. Be in residence for at least three years;
6. Complete a dissertation that clearly advances understanding of the subject it considers.

All students must also complete any other Graduate School of Arts and Sciences degree requirements; see Degree Requirements under Policies and Regulations.

The normal time for completion of the Ph.D. program is five years. Requirement (1) normally includes basic courses in algebra, analysis, and topology. A sequence of
three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term. All qualifying examinations must be passed by the end of the second year. There is no limit to the number of times that students can take the exams, and so they are encouraged to take them as soon as possible.

The dissertation prospectus should be submitted during the third year.

The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1) – (5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 991, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

HONORS REQUIREMENT
Students must meet the Graduate School’s Honors requirement by the end of the fourth term of full-time study.

TEACHING
Teaching experience is integral to graduate education at Yale. Therefore, teaching is required of all graduate students, typically one term per year. Generally, first-year students work as coaches for calculus classes, meeting with small discussion sections of undergraduates. Second-year students often work as teaching assistants for a linear algebra class (MATH 222, MATH 225, or MATH 226), real analysis (MATH 255 or MATH 256), or discrete mathematics (MATH 244); duties usually include holding office hours or leading discussion sections.

In the spring of their second year, graduate students attend the Lang Teaching Seminar (MATH 827). In this lunch seminar, experienced faculty help students understand the challenges of teaching and prepare students to lead their own section of calculus in the following year and beyond.

Students who require additional support from the Graduate School after the fifth year of study must teach additional terms, if needed.

MASTER’S DEGREES
M.Phil. See Degree Requirements under Policies and Regulations.

M.S. (en route to the Ph.D.) A student must complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year. The M.S. degree is conferred only en route to the Ph.D.; there is no terminal master’s degree program in Mathematics.

COURSES

MATH 500a, Algebra  Junliang Shen
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings such as localization and integral extension, connections to algebraic geometry, categories, functors and functor morphisms, tensor
product and Hom functors, and projective modules. Other topics may be discussed at the instructor's discretion. Prerequisites: MATH 350 and MATH 370.

MATH 515b, Intermediate Complex Analysis  Ebru Toprak
Topics may include argument principle, Rouche's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem; functions of finite order, Hadamard's theorem, meromorphic functions; Mittag-Leffler's theorem, subharmonic functions.

MATH 520a, Measure Theory and Integration  Charles Smart
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 522a / CPSC 644a, Geometric and Topological Methods in Machine Learning  Smita Krishnaswamy and Ian Adelstein
This course provides an introduction to geometric and topological methods in data science. Our starting point is the manifold hypothesis: that high dimensional data live on or near a much lower dimensional smooth manifold. We introduce tools to study the geometric and topological properties of this manifold in order to reveal relevant features and organization of the data. Topics include: metric space structures, curvature, geodesics, diffusions maps, eigenmaps, geometric model spaces, gradient descent, data embeddings and projections, and topological data analysis (TDA) in the form of persistence homology and their associated “barcodes.” We see applications of these methods in a variety of data types. Prerequisites: MATH 225 or 226; MATH 255 or 256; MATH 302 and CPSC 112. Students who completed MATH 231 or 250 may substitute another analysis course level 300 or above in place of MATH 302. Familiarity with algorithms/programming is beneficial.

MATH 525b, Introduction to Functional Analysis  Wilhelm Schlag
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 520.

MATH 526a, Introduction to Differentiable Manifolds  Jiewon Park
This is an introduction to the general theory of smooth manifolds, developing tools for use elsewhere in mathematics. A rough plan of topics (with the later ones as time permits) includes (1) manifolds, tangent spaces, vector fields and flows; (2) natural examples, submanifolds, quotient manifolds, fibrations, foliations; (3) vector and tensor bundles, differential forms; (4) Lie derivatives, Lie algebras and groups; (5) embedding, immersions and transversality; (6) Sard's theorem, degree and intersection. Prerequisites: some multivariable calculus, linear algebra, and topology.

MATH 533b, Introduction to Representation Theory  Ivan Loseu
An introduction to basic ideas and methods of representation theory of finite groups and Lie groups. Examples include permutation groups and general linear groups. Connections with symmetric functions, geometry, and physics.

MATH 536b, Combinatorics  Yakov Kononov
Combinatorics is a relatively new and very active area of mathematics, focusing on the study of discrete systems. It has applications in all areas of mathematics, from probability and physics to representation theory and algebraic geometry. It also plays an essential role in computing and data science. The course covers the basic topics of
combinatorics, including generating functions, partitions, symmetric polynomials, random matrices, probabilistic methods, additive combinatorics, and graph theory. Prerequisite: Math 345.

**MATH 544a, Introduction to Algebraic Topology**  Subhadip Dey
This is a one-term graduate introductory course in algebraic topology. We discuss algebraic and combinatorial tools used by topologists to encode information about topological spaces. Broadly speaking, we study the fundamental group of a space, its homology, and its cohomology. While focusing on the basic properties of these invariants, methods of computation, and many examples, we also see applications toward proving classical results. These include the Brouwer fixed-point theorem, the Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher’s *Algebraic Topology*, which is available for free on his website.

**MATH 573a, Algebraic Number Theory**  Alexander Goncharov
Structure of fields of algebraic numbers (solutions of polynomial equations with integer coefficients) and their rings of integers; prime decomposition of ideals and finiteness of the ideal class group; completions and ramification; adeles and ideles; zeta functions.

**MATH 621a, Introduction to Ergodic Theory and Dynamical Systems**  Sebastian Hurtado-Salazar
During the last fifty years, the study of ergodic theory and dynamical systems had multiple applications in diverse fields such as physics, geometry and number theory. This course is an introductory course to these subjects. Among the prerequisites are measure theory and some basic notions in differential geometry and functional analysis. Topics covered: basic notions and examples of ergodic systems; ergodic theorems, including the mean ergodic theorem, pointwise ergodic theorem and the multiplicative ergodic theorem; entropy; uniform and non-uniform hyperbolic dynamics, including topics about Anosov diffeomorphisms and geodesic flows in negative curvature. Other possible topics we might cover include the Ledrappier Young entropy formula, the KAM linearization method, and basics of thermodynamics formalism.

**MATH 627a, Probability Theory**  Staff
Probability theory concerns rigorous definition and study of random phenomena. Besides being an engaging field of study by itself, or serving as a basis for statistics and other applications, it touches on many parts of Mathematics, such as analysis, combinatorics, statistical mechanics, dynamics, etc. It is also important for mathematical understanding of Physics. The interplay between probability theory and other fields of study has growing influence on the most recent ideas and developments of research. The aim of this course is to deepen probabilistic education and to prepare for work in the areas of Mathematics related to probability theory. The main topics covered include (time permitting): axiomatic approach and introduction to random structures, limit theorems, random walks and martingales, random point processes, stochastic processes, Brownian motion and basics of Ito calculus, Markov chains and diffusions, introduction to random matrices and to statistical mechanics. Prerequisite: A course in probability or in measure theory, equivalent to Math 241, Math 320, or Math 330. Some knowledge of measure theory and probabilistic intuition will be assumed.
MATH 628a, Harmonic Analysis  Wilhelm Schlag
We focus on the analysis on Lie groups, more specifically matrix groups such as SU(2) and SO(3). We develop the representation theory of these groups as needed for the Fourier transform (Peter-Weyl theorem). Applications to spherical harmonics and the heat equation are given. The class text is Jacques Faraut's Cambridge studies in advanced mathematics 110, “Analysis on Lie groups: an introduction”. It limits itself to matrix groups, and contains many exercises. Homework problems are assigned and the grade is based on the homework. Prerequisites are real analysis including measure theory and linear algebra. Some previous exposure to Fourier series on the circle would be helpful. Group theory is also a prerequisite but not necessarily Lie groups.

MATH 666a / AMTH 666a / ASTR 666a / EPS 666a, Classical Statistical Thermodynamics  John Wettlaufer
Classical thermodynamics is derived from statistical thermodynamics. Using the multi-particle nature of physical systems, we derive ergodicity, the central limit theorem, and the elemental description of the second law of thermodynamics. We then develop kinetics, transport theory, and reciprocity from the linear thermodynamics of irreversible processes. Topics of focus include Onsager reciprocal relations, the Fokker-Planck equation, stability in the sense of Lyapunov, and time invariance symmetry. We explore phenomena that are of direct relevance to astrophysical and geophysical settings. No quantum mechanics is necessary as a prerequisite.

MATH 675a / AMTH 675a, Numerical Methods for Partial Differential Equations  Vladimir Rokhlin

MATH 701a / AMTH 701a, Topics in Analysis  Peter Jones
This course provides an introduction to some topics in harmonic analysis and probability. Starting with basic dyadic analysis, we use this to give a short introduction to stochastic processes. We then give an introduction to quasiconformal mappings and results concerning random Jordan curves in R2. The main theorem discussed at the end of the course is contained in K. Astala, P. Jones, A. Kupiainen, E. Saksman, “Random Conformal Weldings,” *Acta Mathematica* 207 (2011): 203–254. Some of the topics to be covered are: dyadic grids, maximal functions, and domain decomposition; Haar wavelet analysis, square functions, and Lp estimates; positive measures and product formulas, dyadic earth mover distances; wavelets and applications to function spaces;
probability theory in the dyadic setting and the martingale convergence theorem; random walk, Brownian motion (via Haar functions) and introduction to stochastic processes, Feynman-Kac formalism; Brownian motion and relations to L2. Other topics covered depend on students’ interests and could include: the Johnson-Lindenstrauss theorem and relations to random Gaussian vectors; the Gaussian Free Field and Kahane’s theorem on exponentenation of the GFF; multiscale estimates for Kahane’s theorem; degenerate QC mappings and applications related to Kahane’s theorem on the GFF. A background in basic graduate-level analysis (e.g., MATH 320 and MATH 325) is assumed, though most of the material can be understood by anyone with an understanding of Lebesgue measure.

**MATH 710b / AMTH 710b, Harmonic Analysis on Graphs and Applications**  Ronald Coifman

This class covers basic methods of classical harmonic analysis that can be carried over to graphs and data analysis. We cover the fundamentals of nonlinear Fourier analysis, including functional approximations in high dimensions. We intend to cover foundational material useful for data organization and geometries.

**MATH 718a, Enumerative Geometry and Integrable Systems**  Yakov Kononov

Vertex functions are important generalizations of q-hypergeometric functions. They are related to the theory of quasimaps to Nakajima quiver varieties. We discuss such theories, quantum difference equations, shifts and wall-crossing operators, 3D mirror symmetry, and relation to the algebraic Bethe-ansatz. Prerequisites: basic algebraic geometry and representation theory.

**MATH 719a, Introduction to Anosov Subgroups**  Subhadip Dey

Anosov subgroups, introduced by Labourie and Guichard-Wienhard, are a special kind of discrete subgroups of semi-simple Lie groups which exhibit the geometrical and dynamical properties of convex-cocompact Kleinian groups. This course studies the Anosov subgroups: The main focus is to understand different characterizations Anosov subgroups, mainly those given by Kapovich-Leeb-Porti and Bochi-Potrie-Sambarino. The main topics include: symmetric spaces and the associated flag-manifolds, limit sets of Anosov subgroups, higher rank Morse Lemma and its consequences, characterizations of Anosov subgroups, and thickening of limit sets and the domains of discontinuity of Anosov subgroups. If time permits, we discuss the relatively Anosov subgroups, a more general class of discrete subgroups which extend the geometrically-finite Kleinian groups. Familiarity with the geometry of hyperbolic spaces and Kleinian groups will be helpful.

**MATH 720a, Topics in Representation theory**  Ivan Loseu

The course introduces students to modern results and techniques in the Geometric representation theory and their applications to the representation theory of semisimple Lie algebras.

**MATH 721a, Topics in Homogeneous Dynamics**  Hee Oh

We discuss various topics on homogeneous dynamics and discrete subgroups of Lie groups.

**MATH 722a, Minimal Surfaces**  Lu Wang

We discuss basic topics in minimal surfaces: the first and second variation of area, stability, Bernstein Theorem, Schoen-Simon-Yau curvature estimate, solution to the

**MATH 723a, Random Structures** Van Vu
This course discusses some of the most important random structures, especially from the statistical/computing science point of view. These include several models of random graphs and random matrices.

**MATH 827b, Lang Teaching Seminar** Miki Havlickova
This course prepares graduate students for teaching calculus classes. It is a mix of theory and practice, with topics such as preparing classes, presenting new concepts, choosing examples, encouraging student participation, grading fairly and effectively, implementing active learning strategies, and giving and receiving feedback. Open only to mathematics graduate students in their second year.

**MATH 991a / CPSC 991a, Ethical Conduct of Research** Holly Rushmeier