MATHEMATICS

219 Prospect St
http://math.yale.edu
M.S., M.Phil., Ph.D.

Chair
Wilhelm Schlag

Director of Graduate Studies
Van Vu

Professors Richard Beals (*Emeritus*), Jeffrey Brock, Andrew Casson (*Emeritus*), Ronald Coifman, Igor Frenkel, Howard Garland (*Emeritus*), Alexander Goncharov, Roger Howe (*Emeritus*), Peter Jones, Richard Kenyon, Ivan Loseu, Gregory Margulis (*Emeritus*), Yair Minsky, Vincent Moncrief (*Physics*), Andrew Neitzke, Hee Oh, Nicholas Read (*Physics*; *Applied Physics*), Vladimir Rokhlin (*Computer Science*), Wilhelm Schlag, John Schotland, George Seligman (*Emeritus*), Charles Smart, Daniel Spielman (*Computer Science*), Van Vu, Lu Wang, John Wettlaufer (*Earth and Planetary Sciences*; *Physics*), Gregg Zuckerman (*Emeritus*)

Assistant Professor Junliang Shen

FIELDS OF STUDY

Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; probability; random matrix theory, Kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; Langlands program; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; probabilistic combinatorics; additive combinatorics; and spectral graph theory.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE

In order to qualify for the Mathematics Ph.D., all students are required to:

- complete eight term courses at the graduate level, at least two with Honors grades;
- pass qualifying examinations on their general mathematical knowledge;
- submit a dissertation prospectus;
- participate in the instruction of undergraduates;
- be in residence for at least three years; and
- complete a dissertation that clearly advances understanding of the subject it considers.

All students must also complete any other Graduate School of Arts and Sciences degree requirements; see Degree Requirements under Policies and Regulations.

The normal time for completion of the Ph.D. program is five years. Requirement (1) normally includes basic courses in algebra, analysis, and topology. A sequence of three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term. All qualifying examinations must be passed by the end of the second year. There is no limit to the number of times that students can take the exams, and so they are encouraged to take them as soon as possible.

The dissertation prospectus should be submitted during the third year.

The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1)–(5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 991, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

HONORS REQUIREMENT

Students must meet the Graduate School's Honors requirement by the end of the fourth term of full-time study.

TEACHING

Teaching experience is integral to graduate education at Yale. Therefore, teaching is required of all graduate students, typically one term per year. Generally, first-year students work as coaches for calculus classes, meeting with small discussion sections of undergraduates. Second-year students often work as teaching assistants for a linear algebra class (MATH 222, MATH 225, or MATH 226), real analysis (MATH 255 or MATH 256), or discrete mathematics (MATH 244); duties usually include holding office hours or leading discussion sections.

In the spring of their second year, graduate students attend the Lang Teaching Seminar (MATH 827). In this lunch seminar, experienced faculty help students understand the challenges of teaching and prepare students to lead their own section of calculus in the following year and beyond.

Students who require additional support from the Graduate School after the fifth year of study must teach additional terms, if needed.

MASTER'S DEGREES

M.Phil. See Degree Requirements under Policies and Regulations.

M.S. Students who withdraw from the Ph.D. program may be eligible to receive the M.S. degree if they have met the requirements and have not already received the M.Phil. degree. For the M.S., students must successfully complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year.

COURSES

MATH 500a, Algebra Junliang Shen
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings such as localization and integral extension,

connections to algebraic geometry, categories, functors and functor morphisms, tensor product and Hom functors, and projective modules. Other topics may be discussed at the instructor's discretion. Prerequisites: MATH 350 and MATH 370.

MATH 515b, Intermediate Complex Analysis Alexander Goncharov
Topics may include argument principle, Rouché's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem; functions of finite order, Hadamard's theorem, meromorphic functions; Mittag-Leffler's theorem, subharmonic functions.

MATH 520a, Measure Theory and Integration Charles Smart
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 525b, Introduction to Functional Analysis Hanwen Zhang
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 520.

MATH 526a, Introduction to Differentiable Manifolds Tamunonye Cheetham-West
This is an introduction to the general theory of smooth manifolds, developing tools for use elsewhere in mathematics. A rough plan of topics (with the later ones as time permits) includes (1) manifolds, tangent spaces, vector fields and flows; (2) natural examples, submanifolds, quotient manifolds, fibrations, foliations; (3) vector and tensor bundles, differential forms; (4) Lie derivatives, Lie algebras and groups; (5) embedding, immersions and transversality; (6) Sard's theorem, degree and intersection. Prerequisites: some multivariable calculus, linear algebra, and topology.

MATH 533b, Introduction to Representation Theory Igor Frenkel
An introduction to basic ideas and methods of representation theory of finite groups and Lie groups. Examples include permutation groups and general linear groups. Connections with symmetric functions, geometry, and physics.

MATH 536b, Combinatorics Nicholas Ovenhouse
Combinatorics is a relatively new and very active area of mathematics, focusing on the study of discrete systems. It has applications in all areas of mathematics, from probability and physics to representation theory and algebraic geometry. It also plays an essential role in computing and data science. The course covers the basic topics of combinatorics, including generating functions, partitions, symmetric polynomials, random matrices, probabilistic methods, additive combinatorics, and graph theory. Prerequisite: Math 345.

MATH 544a, Introduction to Algebraic Topology Alexander Goncharov
This is a one-term graduate introductory course in algebraic topology. We discuss algebraic and combinatorial tools used by topologists to encode information about topological spaces. Broadly speaking, we study the fundamental group of a space, its homology, and its cohomology. While focusing on the basic properties of these invariants, methods of computation, and many examples, we also see applications toward proving classical results. These include the Brouwer fixed-point theorem, the Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher's *Algebraic Topology,* which is available for free on his website.

MATH 640b / AMTH 640b / CPSC 640b, Topics in Numerical Computation
Vladimir Rokhlin
This course discusses several areas of numerical computing that often cause difficulties to non-numericists, from the ever-present issue of condition numbers and ill-posedness to the algorithms of numerical linear algebra to the reliability of numerical software. The course also provides a brief introduction to "fast" algorithms and their interactions with modern hardware environments. The course is addressed to Computer Science graduate students who do not necessarily specialize in numerical computation; it assumes the understanding of calculus and linear algebra and familiarity with (or willingness to learn) either C or FORTRAN. Its purpose is to prepare students for using elementary numerical techniques when and if the need arises.

MATH 665a, Topics in Quantum Algebra Minh-Tam Trinh
We discuss the relationship between representations of linear groups over finite and p-adic fields, a part of Lie theory, and isotopy invariants of knot and links, a part of geometric topology. The bridge is the theory of Hecke algebras and their cocenters. We begin with the classical references, potentially including works of Jones, Deligne–Lusztig, and Macdonald and, working our way through the categorification program of Frenkel, Khovanov, and others, aim to arrive at recent works about double affine Hecke algebras and algebraic links.

MATH 666a / AMTH 666a / ASTR 666a / EPS 666a, Classical Statistical Thermodynamics John Wettlaufer
Classical thermodynamics is derived from statistical thermodynamics. Using the multi-particle nature of physical systems, we derive ergodicity, the central limit theorem, and the elemental description of the second law of thermodynamics. We then develop kinetics, the origin of diffusion, transport theory, and reciprocity from the linear thermodynamics of irreversible processes. Topics of focus include Onsager reciprocal relations, the Fokker-Planck and Cahn-Hilliard equations, stability in the sense of Lyapunov, time invariance symmetry and maximum principles. We explore phenomena cross a range of problems in science and engineering. Prerequisites for Yale College students: PHYS 301, PHYS 410, MATH 246 or similar and/or permission of instructor.

MATH 675a / AMTH 675a, Numerical Methods for Partial Differential Equations
Vladimir Rokhlin
(1) Review of the classical qualitative theory of ODEs; (2) Cauchy problem. Elementary numerical methods: Euler, Runge-Kutta, predictor-corrector. Stiff systems of ODEs: definition and associated difficulties, implicit Euler, Crank-Nicolson, barrier theorems. Richardson extrapolation and deferred corrections; (3) Boundary value problems. Elementary theory: finite differences, finite elements, abstract formulation and related spaces, integral formulations and associated numerical tools, nonlinear problems; (4) Partial differential equations (PDEs). Introduction: counterexamples, Cauchy–Kowalevski theorem, classification of second-order PDEs, separation of variables; (5) Numerical methods for elliptic PDEs. Finite differences, finite elements, Richardson and deferred corrections, Lippmann–Schwinger equation and associated numerical tools, classical potential theory, "fast" algorithms; (6) Numerical methods for parabolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, integral formulations and related numerical tools; (7) Numerical methods for hyperbolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, time-invariant problems and Fourier transform.

MATH 680a, Fourier Analysis and PDEs Wilhelm Schlag
This course covers some of the finer techniques in Fourier analysis relevant to nonlinear PDEs. We cover some multilinear estimates of the Coifman-Meyer type and the related para-differential calculus. Classical results in time-frequency analysis including the Beurling-Malliavin theorem and its ramifications might also be included. Students should have taken multivariable calculus, Math 305 and 325. In addition, exposure to complex analysis is recommended as well.

MATH 685a, Topics in Representation Theory Igor Frenkel
The course is dedicated to modern directions in representation theory developed in the last several decades and to related subjects. The program largely depends on the interests of the audience. The participants are encouraged to give presentations on the representation theory topics related to their research. The directions covered may include (but are not limited to): representations of infinite-dimensional Lie algebras including Virasoro algebra and quaternionic analysis, vertex operator algebras, geometric representation theory, categorification and Khovanov homology, cluster algebras, and quantum Teichmuller spaces.

MATH 690a, Introduction to Quantum Invariants of Knots and Three Manifolds Ka Ho Wong
This course is an introduction to quantum invariants of knots and three manifolds and their relationships with hyperbolic geometry. Topics include the skein-theoretic constructions of the Jones and colored Jones polynomials, the Witten-Reshetikhin-Turaev invariants, the Turaev-Viro invariants, and their underlying Topological Quantum Field Theory. Interactions between quantum topology and hyperbolic geometry, such as the Kashaev-Murakami-Murakami volume conjecture and its generalizations, are also discussed in this course.

MATH 695a, High-Dimensional Probability Pei-Chun Su
The course introduces the fundamental concepts and advanced techniques of concentration inequalities and classical results like Hoeffding's and Chernoff's inequalities alongside modern developments such as the matrix Bernstein's inequality. Discover the potency of stochastic processes through Slepian's, Sudakov's, and Dudley's inequalities, as well as generic chaining and bounds rooted in VC dimension.

MATH 827b, Lang Teaching Seminar Brett Smith and Bailey Heath
This course prepares graduate students for teaching calculus classes. It is a mix of theory and practice, with topics such as preparing classes, presenting new concepts, choosing examples, encouraging student participation, grading fairly and effectively, implementing active learning strategies, and giving and receiving feedback. Open only to mathematics graduate students in their second year.

MATH 991a / CPSC 991a, Ethical Conduct of Research Inyoung Shin
This course forms a vital part of research ethics training, aiming to instill moral research codes in graduate students of computer science, math, and applied math. By delving into case studies and real-life examples related to research misconduct, students grasp core ethical principles in research and academia. The course also offers an opportunity to explore the societal impacts of research in computer science, math, and applied math. This course is designed specifically for first-year graduate students in computer science, applied math, and math. Successful completion of the course necessitates in-person attendance on eight occasions; virtual participation

does not fulfill this requirement. In cases where illness, job interviews, or unforeseen circumstances prevent attendance, makeup sessions are offered. o Course cr