Mathematics

10 Hillhouse Avenue, 203.432.7058
http://math.yale.edu
M.S., M.Phil., Ph.D.

Chair
Yair Minsky

Director of Graduate Studies
Alexander Goncharov [F]
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Professors
Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Gil Kalai (Adjunct), Ivan Losev, Alexander Lubotzky (Adjunct), Gregory Margulis, Yair Minsky, Vincent Moncrief (Physics), Hee Oh, Sam Payne, Nicholas Read (Physics; Applied Physics), Vladimir Rokhlin (Computer Science), Wilhelm Schlag, George Seligman (Emeritus), Daniel Spielman (Computer Science), Van Vu, John Wettlaufer (Geology & Geophysics; Physics), Gregg Zuckerman

Associate Professor
Yifeng Liu

Assistant Professor
Stefan Steinerberger

Fields of Study
Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; geometric analysis; kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; combinatorics and discrete mathematics.

Special Requirements for the Ph.D. Degree
All students are required to: (1) complete eight term courses at the graduate level, at least two with Honors grades; (2) pass qualifying examinations on their general mathematical knowledge; (3) submit a dissertation prospectus; (4) participate in the instruction of undergraduates; (5) be in residence for at least three years; and (6) complete a dissertation that clearly advances understanding of the subject it considers. The normal time for completion of the Ph.D. program is five years. Requirement (1) should be completed by the end of the second year. A sequence of three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term, at intervals of about one month. All qualifying examinations must be taken by the end of the third term. The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1)–(5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 991, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

Honors Requirement
Students must meet the Graduate School's Honors requirement by the end of the fourth term of full-time study.

Teaching
Teaching experience is integral to graduate education at Yale. Therefore, most Mathematics students are required to assist in teaching during five terms. Students in years one and two serve as tutors and graders in undergraduate mathematics courses during one term per year. The department also offers a required teaching practicum in year two. In years three through five, students normally teach one section of calculus or its equivalent during one term per year. Students receiving external fellowships may petition for a waiver of teaching while receiving external funding in place of University funding, but they are still required to teach one section of calculus or its equivalent for a minimum of two terms over the course of their program.

Master's Degrees
M.Phil. In addition to the Graduate School's Degree Requirements (see under Policies and Regulations), a student must undertake a reading program of at least two terms' duration in a specific significant area of mathematics under the supervision of a faculty adviser and demonstrate command of the material studied during the reading period at a level sufficient for teaching and research.

M.S. (en route to the Ph.D.) A student must complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year. The M.S. degree is conferred only en route to the Ph.D.; there is no separate master's program in Mathematics.
COURSES

MATH 500a, Modern Algebra I  Yifeng Liu
A survey of algebraic constructions and theories at a sophisticated level. Topics include categorical language, free groups and other free objects in categories, general theory of rings and modules, artinian rings, and introduction to homological algebra.

MATH 503b, Heat Kernel and Analysis on Manifolds  Stefan Steinerberger
Topics include Laplace operator on Riemannian manifolds, heat equation, maximum principles and regularity theory, spectral properties, the distance function, Gaussian estimates, Davies-Gaffney estimates, Green function, Ultracontractive estimates, pointwise Gaussian estimates. The goal is to go through Heat Kernel and Analysis on Manifolds by Alexander Grigor’yan.

MATH 504b, Geodesic Currents and Counting Problems  Caglar Uyanik
Geodesic currents are measure theoretic generalizations of closed curves on hyperbolic surfaces, and they play an important role, among other things, in the study of Teichmüller spaces and mapping class groups. The goal of this course is to study groundbreaking work of Mirzakhani on counting closed geodesics on hyperbolic surfaces [Mir08, Mir16] and its generalizations [EPS16, ES16, RS17] using geodesic currents. We first review background material such as hyperbolic geometry, mapping class groups, Teichmüller and moduli spaces, measured laminations, and train tracks. We then move to geodesic currents and discuss applications to counting problems on hyperbolic surfaces. The main sources for the first part are CB88, FM11, and PH16, and the second part mainly draws from Bon86, Bon88, Miro8, Mir16, EPS16, ES16, EU18, and RS17.
Prerequisite: the contents of Algebraic Topology 1.

MATH 505a, Mathematics of Two-Dimensional Quantum Gravity (and Topological String Theory)  PhilSang Yoo
Kontsevich’s proof of Witten’s conjecture is discussed in some detail, including, at the maximum, the string theory approach through BCOV theory of Costello-Li. Likely the course ends up somewhere in the middle, covering some of Baramnikov-Kontsevich, K. Saito, Givental, and/or Dubrovin after dealing with Witten-Kontsevich theory. Main topics include moduli of curves, Teichmüller theory, combinatorics of graphs on surfaces, integrable hierarchy, two-dimensional TQFT, and mirror symmetry.

MATH 506a, Cocycles, Lyapunov Exponents, and Spectral Theory  Wilhelm Schlag
We develop cocycles over an ergodic base, prove the basic ergodic theorems (Kingman), and discuss Lyapunov exponents, Osseldt’s theorem, and Furstenberg’s theorem in the random case, as well as its higher-dimensional version. We connect these results to the spectral theory of Schrödinger operators with potentials defined by an ergodic process, introduce the avalanche principle, and establish Hölder regularity of the Lyapunov exponent in the energy.

MATH 507b, Infinite-Dimensional Lie Algebras and Applications  Oleksandr Tsymbaliuk
The basic course studying infinite-dimensional Lie algebras, the key two examples of which are Virasoro Lie algebra and affine Kac-Moody algebras. Their theory as well as various applications to other areas are discussed.

MATH 520a, Measure Theory and Integration  Arie Levit
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 544a, Introduction to Algebraic Topology I  Yair Minsky
A one-semester graduate introductory course in algebraic topology. We discuss algebraic and combinatorial tools used by topologists to encode information about topological spaces. Broadly speaking, we study the fundamental group of a space, its homology, and its cohomology. While focusing on the basic properties of these invariants, methods of computation, and many examples, we also see applications toward proving classical results. These include the Brouwer fixed-point theorem, the Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher’s Algebraic Topology, which is available for free on his website.

MATH 545b, Introduction to Algebraic Topology II  Staff
Course Description: This course will explore some of the major themes in arithmetic geometry, i.e., the study of algebraic varieties over arbitrary fields. Topics will include quadratic forms and the Brauer group, Galois cohomology and descent, principal homogeneous spaces and local-global principles, as well as the existence of rational points and rational parametrizations. Of particular interest will be varieties over finite fields, valued fields, real closed fields, Ci-fields, number fields, and functions fields. Recurring objects of study throughout the course will be quadric hypersurfaces and rational surfaces from an arithmetic and geometry point of view. Some prior experience with algebra, Galois theory, and algebraic geometry will be very helpful but not strictly necessary. Can be taken in conjunction with a beginning algebraic geometry course.

MATH 566a, Classical Statistical Thermodynamics  John Wettlaufer
Classical thermodynamics is derived from statistical thermodynamics. Using the multi-particle nature of physical systems, we derive ergodicity, the central limit theorem, and the elemental description of the second law of thermodynamics. We then develop kinetics, transport theory, and reciprocity from the linear thermodynamics of irreversible processes. Topics of focus include Onsager reciprocal relations, the Fokker-Planck equation, stability in the sense of Lyapunov, and time invariance symmetry. We explore phenomena that are of direct relevance to astrophysical and geophysical settings. No quantum mechanics is necessary as a prerequisite.

MATH 991a / CPSC 991a, Ethical Conduct of Research  Alexander Goncharov
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