MATHEMATICS (MATH)

MATH 106b, The Shape of Space  Ian Adelstein
This course provides an introduction to mathematical thinking through ideas in geometry and graph theory. Traditional lecture, worksheets, discussion, group work, and classroom activities all contribute to a dynamic learning experience. The course follows a historical narrative, starting from antiquity, to understand the foundations of mathematical thought. An axiomatic approach to geometry affords students the opportunity to construct proofs of classical theorems. The basics of graph theory are introduced in order to explore real world problems such as map coloring and bridge crossing. The ancient Greek method of exhaustion previews a discussion of the integral, and from here we explore the beautiful relationship between the geometry and topology of graphs, polyhedra, and surfaces. Throughout the course students build their mathematical and geometric intuition through problem solving and exercises in geometric imagining. Permission of instructor required. Enrollment limited to 25 students who have not previously taken a high school or college calculus course. QR

* MATH 107a, Mathematics in the Real World  Rachel Diethorn
The use of mathematics to address real-world problems. Applications of exponential functions to compound interest and population growth; geometric series in mortgage payments, amortization of loans, present value of money, and drug doses and blood levels; basic probability, Bayes's rule, and false positives in drug testing; elements of logic. Permission of instructor required. Enrollment limited to 25 students who have not previously taken a high school or college calculus course. QR

MATH 108b, Estimation and Error  Staff
A problem-based investigation of basic mathematical principles and techniques that help make sense of the world. Estimation, order of magnitude, approximation and error, counting, units, scaling, measurement, variation, simple modeling. Applications to demographics, geology, ecology, finance, and other fields. Emphasis on both the practical and the philosophical implications of the mathematics. Permission of instructor required. Enrollment limited to 25 students who have not previously taken a high school or college calculus course. QR

* MATH 110a, Introduction to Functions and Calculus I  Robert McDonald
Comprehensive review of precalculus, limits, differentiation and the evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of MATH 110 and 111 is equivalent to MATH 112. No prior acquaintance with calculus is assumed; some knowledge of algebra and precalculus mathematics is helpful. Placement into MATH 110 on the Mathematics placement exam is required. Enrollment in MATH 110 is through preference selection. QR

* MATH 111b, Introduction to Functions and Calculus II  Staff
Continuation of MATH 110. Comprehensive review of precalculus, limits, differentiation and evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of both MATH 110 and 111 is equivalent to MATH 112. Prerequisite: MATH 110. Enrollment in MATH 111 is through preference selection. QR

* MATH 112a or b, Calculus of Functions of One Variable I  Staff
Limits and their properties. Definitions and some techniques of differentiation and the evaluation of definite integrals, with applications. Use of the software package Mathematica to illustrate concepts. Placement into MATH 112 on the Mathematics placement exam is required. No prior acquaintance with calculus or computing assumed. May not be taken after MATH 111. Enrollment in MATH 112 is through preference selection. QR

* MATH 115a or b, Calculus of Functions of One Variable II  Staff
A continuation of MATH 112. Applications of integration, with some formal techniques and numerical methods. Improper integrals, approximation of functions by polynomials, infinite series. Prerequisite: MATH 111 or MATH 112, or placement into MATH 115 on the Mathematics placement exam. May not be taken after MATH 116. Enrollment in MATH 115 is through preference selection. QR

* MATH 116a, Mathematical Models in the Biosciences I: Calculus Techniques  John Hall
Techniques and applications of integration, approximation of functions by polynomials, modeling by differential equations. Introduction to topics in mathematical modeling that are applicable to biological systems. Discrete and continuous models of population, neural, and cardiac dynamics. Stability of fixed points and limit cycles of differential equations. Prerequisite: MATH 112, or placement into MATH 115/116 on the Mathematics placement exam. May not be taken after MATH 115. QR

* MATH 118a or b, Introduction to Functions of Several Variables  Staff
A combination of linear algebra and differential calculus of several variables. Matrix representation of linear equations, Gauss elimination, vector spaces, independence, basis and dimension, projections, least squares approximation, and orthogonality. Three-dimensional geometry, functions of two and three variables, level curves and surfaces, partial derivatives, maxima and minima, and optimization. Intended for students in the social sciences, especially Economics. May not be taken after MATH 120 or 222. Prerequisite: MATH 112. QR

* MATH 120a or b, Calculus of Functions of Several Variables  Staff
Analytic geometry in three dimensions, using vectors. Real-valued functions of two and three variables, partial derivatives, gradient and directional derivatives, level curves and surfaces, maxima and minima. Parametrized curves in space, motion in space, line integrals;
applications. Multiple integrals, with applications. Divergence and curl. The theorems of Green, Stokes, and Gauss. Prerequisite: MATH 115 or 116, or placement into MATH 120 on the Mathematics placement exam. May not be taken after MATH 121. Enrollment in MATH 120 is through preference selection. QR

* MATH 121b, Mathematical Models in the Biosciences II: Advanced Techniques  John Hall
Mathematical modeling for the biosciences, with a strong focus on multivariable calculus techniques. Applications may include epidemiological models, mathematical foundations of virus and antiviral dynamics, ion channel models and cardiac arrhythmias, and evolutionary models of disease. Prerequisite: MATH 115 or 116, or placement into MATH 120/121 on the Mathematics placement exam. May not be taken after MATH 120. QR

MATH 160b / AMTH 160b / S&DS 160b, The Structure of Networks  Staff
Network structures and network dynamics described through examples and applications ranging from marketing to epidemics and the world climate. Study of social and biological networks as well as networks in the humanities. Mathematical graphs provide a simple common language to describe the variety of networks and their properties. QR

MATH 222a or b / AMTH 222a or b, Linear Algebra with Applications  Staff

MATH 225a or b, Linear Algebra  Staff
An introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, inner product spaces, spectral theorem. The course focuses on conceptual understanding and serves as an introduction to writing mathematical proofs. For an approach focused on applications rather than proofs, consider MATH 222. Prerequisite: MATH 115 or equivalent. May not be taken after MATH 222, 226, or 231. QR

* MATH 226a, Linear Algebra (Intensive)  Kevin O'Neill
A fast-paced introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, inner product spaces, spectral theorem. Topics are covered at a deeper level than in MATH 225, and additional topics may be covered, for example canonical forms or the classical groups. The course focuses on conceptual understanding. Familiarity with writing mathematical proofs is recommended. For a less intensive course, consider MATH 225. For an approach focused on applications, consider MATH 222. Prerequisite: MATH 115 or equivalent. May not be taken after MATH 222, 225, or 231. QR

MATH 240b, Advanced Linear Algebra  Subhadip Dey
The course continues the study of linear algebra from MATH 225 or MATH 230/231. It discusses several aspects of linear algebra that are of crucial importance for the subject and its applications to abstract algebra, geometry and number theory. Topics include generalized eigenspaces and Jordan normal form theorem, dual vector spaces, bilinear and hermitian forms, symmetric and hermitian operators, Hom spaces and tensor products. After MATH 225 or 226 or 231.

MATH 241a / S&DS 241a, Probability Theory  Yihong Wu
Introduction to probability theory. Topics include probability spaces, random variables, expectations and probabilities, conditional probability, independence, discrete and continuous distributions, central limit theorem, Markov chains, and probabilistic modeling. After or concurrently with MATH 120 or equivalent. QR

MATH 242b / S&DS 242b, Theory of Statistics  Andrew Barron and William Brinda
Study of the principles of statistical analysis. Topics include maximum likelihood, sampling distributions, estimation, confidence intervals, tests of significance, regression, analysis of variance, and the method of least squares. Some statistical computing. After S&DS 241 and concurrently with or after MATH 222 or 225, or equivalents. QR

MATH 244a or b / AMTH 244a or b, Discrete Mathematics  Staff
Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients, Stirling numbers. Properties of finite set systems. Recommended preparation: MATH 115 or equivalent. QR

MATH 246a or b, Ordinary Differential Equations  Staff
First-order equations, second-order equations, linear systems with constant coefficients. Numerical solution methods. Geometric and algebraic properties of differential equations. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or 226 or equivalent. QR

MATH 247b / AMTH 247b, Intro to Partial Differential Equations  Erik Hiltunen
Introduction to partial differential equations, wave equation, Laplace's equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225 or 226, MATH 246 or ENAS 194 or equivalents. QR

MATH 251b / EENG 434b / S&DS 351b, Stochastic Processes  Staff
Applications chosen from communications, networking, image reconstruction, Bayesian statistics, finance, probabilistic analysis of algorithms, and genetics and evolution. Prerequisite: S&DS 241 or equivalent. QR

MATH 255a or b, Analysis 1  Staff
Introduction to Analysis. Properties of real numbers, limits, convergence of sequences and series. Power series, Taylor series, and the classical functions. Differentiation and Integration. Metric spaces. The course focuses on conceptual understanding and serves as an introduction to writing mathematical proofs. Prerequisite: MATH 115 or equivalent. MATH 225 or 226 is recommended but not required. May not be taken after MATH 256, 300, or 301. QR

MATH 256b, Analysis 1 (Intensive)  Yair Minsky
Fast-paced introduction to Analysis. Properties of real numbers, limits, convergence of sequences and series. Power series, Taylor series, and the classical functions. Differentiation and Integration. Metric spaces. The course focuses on conceptual understanding. Familiarity with writing mathematical proofs is assumed, and is further developed in the course. Prerequisite: MATH 115 or equivalent. MATH 225 or 226 is recommended but not required. May not be taken after MATH 255, 300, or 301. QR

MATH 270a, Set Theory  James Barnes
Algebra of sets; finite, countable, and uncountable sets. Cardinal numbers and cardinal arithmetic. Order types and ordinal numbers. The axiom of choice and the well-ordering theorem. After MATH 120 or equivalent. QR

MATH 302a, Vector Analysis and Integration on Manifolds  Andrew Neitzke
A rigorous treatment of the modern toolkit of multivariable calculus. Differentiation and integration in R^n. Inverse function theorem. Fubini's theorem. Multilinear algebra and differential forms. Manifolds in R^n. Generalized Stokes’ Theorem. The course focuses on conceptual structure and proofs, and serves as a gateway to more advanced courses which use the language of manifolds. Prerequisites: MATH 225 or 226, and MATH 255 or 256. QR

MATH 305b, Analysis 2: Lebesgue Integration and Fourier series  Hee Oh
The Lebesgue integral, Fourier series, applications to differential equations. Prerequisites: MATH 225 or 226, and MATH 255 or 256 or 301. With permission of instructor, may be taken after MATH 225 or 226, and MATH 231 or 250. QR

MATH 310a, Introduction to Complex Analysis  Franco Vargas Pallete
An introduction to the theory and applications of functions of a complex variable. Differentiability of complex functions. Complex integration and Cauchy’s theorem. Series expansions. Calculus of residues. Conformal mapping. Prerequisites: MATH 225 or 226 or 231, and MATH 255 or 256 or 230 or 250, and MATH 302 or 120. QR

MATH 315b, Intermediate Complex Analysis  Alexander Goncharov
Continuation of MATH 310. Topics may include argument principle, Rouché’s theorem, Hurwitz theorem, Runge’s theorem, analytic continuation, Schwarz reflection principle, Jensen’s formula, infinite products, Weierstrass theorem. Functions of finite order, Hadamard’s theorem, meromorphic functions. Mittag-Leffler’s theorem, subharmonic functions. After MATH 310. QR RP

MATH 320a, Measure Theory and Integration  Lu Wang
Construction and limit theorems for measures and integrals on general spaces; product measures; L^p spaces; integral representation of linear functionals. After MATH 305 or equivalent. QR RP

MATH 325b, Introduction to Functional Analysis  Wilhelm Schlag
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 320, or after MATH 305 with permission of instructor. QR

MATH 330a / S&DS 400a, Advanced Probability  Sekhar Tatikonda
Measure theoretic probability, conditioning, laws of large numbers, convergence in distribution, characteristic functions, central limit theorems, martingales. Some knowledge of real analysis assumed. QR

MATH 345a, Modern Combinatorics  Andrei Pohoata
Recent developments and important questions in combinatorics. Relations to other areas of mathematics such as analysis, probability, and number theory. Topics include probabilistic method, random graphs, random matrices, pseudorandomness in graph theory and number theory, Szemeredi’s theorem and lemma, and Green-Tao’s theorem. Prerequisite: MATH 244. QR

MATH 350a or b, Introduction to Abstract Algebra  Staff
Group theory, structure of Abelian groups, and applications to number theory. Symmetric groups and linear groups including orthogonal and unitary groups; properties of Euclidean and Hermitian spaces. Some examples of group representations. Modules over Euclidean rings, Jordan and rational canonical forms of a linear transformation. Prerequisites: one term of linear algebra and two terms of proof-based mathematics courses. (For example, MATH 225 and 255, or MATH 225 and 244, or MATH 230 and 231, or MATH 225 and 250.) QR

MATH 354b, Number Theory  Congling Qiu
Prime numbers; quadratic reciprocity law, Gauss sums; finite fields, equations over finite fields; zeta functions. After MATH 350. QR
MATH 360b, Introduction to Lie Groups  Yakov Kononov
Lie groups as the embodiment of the idea of continuous symmetry. The exponential map on matrices and applications; spectral theory; examples and structure of Lie groups and Lie algebras; connections with geometry and physics. After MATH 350 and MATH 302. With permission of instructor, may be taken after Math 350 and Math 231 or 250.  QR

MATH 370b, Fields and Galois Theory  Miki Havlickova
Rings, with emphasis on integral domains and polynomial rings. The theory of fields and Galois theory, including finite fields, solvability of equations by radicals, and the fundamental theorem of algebra. Quadratic forms. After MATH 350.  QR

MATH 373a, Algebraic Number Theory  Congling Qiu
Structure of fields of algebraic numbers (solutions of polynomial equations with integer coefficients) and their rings of integers; prime decomposition of ideals and finiteness of the ideal class group; completions and ramification; adeles and ideles; zeta functions. Prerequisites: MATH 310 and 370.  QR

MATH 380a, Modern Algebra 1  Ivan Loseu
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings, such as localization and integral extension, connections to algebraic geometry, categories, functors and functor morphisms, tensor product and Hom functors, projective modules. Other topics may be discussed at instructor’s discretion. After MATH 350 and 370.  QR

MATH 421b / AMTH 420b, The Mathematics of Data Science  Anna Gilbert
This course aims to be an introduction to the mathematical background that underlies modern data science. The emphasis is on the mathematics but occasional applications are discussed (in particular, no programming skills are required). Covered material may include (but is not limited to) a rigorous treatment of tail bounds in probability, concentration inequalities, the Johnson-Lindenstrauss Lemma as well as fundamentals of random matrices, and spectral graph theory. Prerequisite: MATH 305.  QR, SC

MATH 430b, Introduction to Algebraic Topology  Richard Kenyon
The theory of fundamental groups and covering spaces, with particular reference to two-dimensional manifolds. Prerequisites: MATH 350, and MATH 255 or 256 or 300 or 301.  QR

MATH 440a, Introduction to Algebraic Geometry  Staff
Students develop the theory of algebraic varieties, which are zero sets of polynomial equations, starting with basic commutative algebra and include Groebner basis, chain conditions, and the ideal membership problem. Additional varieties include affine, projective, quasi-projective. We prove Hilbert’s Nullstellensatz (one of the most important theorems of classical algebraic geometry) and discuss different notions of dimension and how they relate. We define maps between varieties, morphisms, rational, and birational maps. The remaining topics are (but are not limited to) singularity theory, normalization and blowups, elimination theory, resultants, and divisors on algebraic varieties. Some focus on computational aspects of algebraic geometry is included. Prerequisites: MATH 350 and MATH 370.  QR

MATH 447a / AMTH 447a, Partial Differential Equations  John Schotland
Introduction to partial differential equations, wave equation, Laplace’s equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 305

MATH 470a or b, Individual Studies  Staff
Individual investigation of an area of mathematics outside of those covered in regular courses, involving directed reading, discussion, and either papers or an examination. A written plan of study approved by the student’s adviser and the director of undergraduate studies is required. The course may normally be elected for only one term.

MATH 475a or b, Senior Essay  Staff
Interested students may write a senior essay under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the director of undergraduate studies at least one semester in advance of the semester in which they plan to write the essay.

* MATH 480a or b, Senior Seminar: Mathematical Topics  Staff
A number of mathematical topics are chosen each term—e.g., differential topology, Lie algebras, mathematical methods in physics—and explored in one section of the seminar. Students give several presentations on the chosen topic. Enrollment limited to seniors majoring in Mathematics, Economics and Mathematics, or Mathematics and Philosophy.

* MATH 481b, Senior Seminar: Topics in Economics and Mathematics  Andrei Pohoata and Dirk Bergemann
A number of topics at the intersection of economics and mathematics are chosen each term—e.g., the theory of networks, market design and equilibrium, information economics and probability — and explored in the seminar. Students present several talks on the chosen topic. This section is devoted to topics of interest to majors in Economics or Mathematics majors, and in particular to students in the joint major Economics and Mathematics. The seminar is co-taught by a member of the Economics Department. Enrollment limited to seniors majoring in Mathematics, Economics and Mathematics, or Mathematics and Philosophy.