

APPLIED MATHEMATICS

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Mathematical models are widely used throughout science and engineering in fields as diverse as physics, bioinformatics, robotics, image processing, and economics. Despite the broad range of applications, there are a few essential techniques used in addressing most problems. The Applied Mathematics major provides a foundation in these mathematical techniques and trains the student to use them in a substantive field of application.

The interdisciplinary major permits a great deal of flexibility in design. It is intended to appeal to students who wish to study the more mathematical aspects of science or engineering as well as those whose primary interest is in mathematics and statistics and who wish to become acquainted with applications. Core courses are drawn from Computer Science, Mathematics, Statistics and Data Science, and Engineering and Applied Science. Courses applying mathematics may be drawn from participating programs in Applied Physics; Astronomy; the biological sciences, including Ecology and Evolutionary Biology, Molecular Biophysics and Biochemistry, and Molecular, Cellular, and Developmental Biology; Chemistry; Economics; the various programs in engineering, including Biomedical, Chemical, Electrical, Environmental, and Mechanical Engineering; Geology and Geophysics; Physics; and Political Science. The Applied Mathematics degree program requires a three-course concentration in a field in which mathematics is used.

Students may pursue a major in Applied Mathematics as one of two majors and can thereby equip themselves with mathematical modeling skills while being fully engaged in a field of application. In this case, the concentration requirement of the Applied Mathematics program is flexible in order to recognize the contribution of the other major. A two-course overlap is permitted in satisfying the requirements of the two majors.

PREREQUISITE AND INTRODUCTORY COURSES

Multivariable calculus and linear algebra are required and should be taken before or during the sophomore year. This requirement may be satisfied by MATH 120 or ENAS 151, and MATH 222 or 225 or equivalents. It may also be satisfied by MATH 230, 231. Computer programming skills are also required and may be acquired by taking ENAS 130, CPSC 100 or 112. Details of individual programs must be worked out in consultation with the director of undergraduate studies, whose signed permission is required.

REQUIREMENTS OF THE MAJOR

The B.A. degree program The program requires eleven term courses beyond the prerequisites, including the senior project, comprising a coherent program:

1. A course in differential equations (ENAS 194 or MATH 246)
2. A course in probability (S&DS 241 or S&DS 238)
3. A course in data analysis (S&DS 361 or S&DS 230)
4. A course in discrete mathematics (AMTH 244 or CPSC 202)
5. Courses in at least three of the following areas: (a) optimization: AMTH 437; (b) probability and statistics: S&DS 242, S&DS 251, S&DS 312, S&DS 364, ECON 136, ENAS 496; (c) partial differential equations and analysis: MATH 247, 250, 260, 300, 301, 310; (d) algorithms and numerical methods: CPSC 365, 440, ENAS 440, 441; (e) graph theory: AMTH 462; (f) mathematical economics: ECON 350, 351; (g) electrical engineering: EENG 397, 436, 442, S&DS 364; (h) data mining and machine learning: S&DS 365, CPSC 445; (i) biological modeling and computation: CPSC 475, BENG 445, ENAS 391; (j) physical sciences: ASTR 320, 420, G&G 322, 323, 421, PHYS 344, 401, 402, 410, 420, 430, 440, 442, 460, APHY 439, 448; (k) engineering: MENG 280, 285, 361, 383, 463, 469, CENG 301, 315
6. At least three courses in a field of concentration involving the application of mathematics to that field, at least two of which are advanced courses. Programs in science, engineering, computer science, statistics, and economics are natural sources of concentration. Alternatively, when two majors are undertaken, if the second major is in a participating program, then, recognizing that there can be an overlap of two courses, the student may take for the remaining course an additional choice relevant to the Applied Mathematics major such as listed in point 5 above or for the B.S. degree below. Details of a student's program to satisfy the concentration requirement must be worked out in consultation with, and approved by, the director of undergraduate studies.

The B.S. degree program In addition to the courses indicated for the B.A. degree, the B.S. degree, which totals fourteen term courses beyond the prerequisites, must also include:

1. Topics in analysis (MATH 300) or introduction to analysis (MATH 301); the course selected may not be counted toward the area requirement for the major (see item 5 above)
2. An additional course selected from the list in item 5 above
3. Another course numbered 300 or higher from the list above, or a course numbered 300 or higher in mathematics, applied mathematics, statistics, or quantitative computer science or engineering, subject to the approval of the director of undergraduate studies

Alternatively, students may petition to receive a B.S. in Applied Mathematics by fulfilling the B.A. requirements in Applied Mathematics and the B.S. requirements in another program.

Credit/D/Fail A maximum of one course credit taken Credit/D/Fail may be counted toward the requirements of the major.

SENIOR REQUIREMENT

Both the B.A. and B.S. degree programs require Senior Seminar and Project (AMTH 490) or a special project completed during senior year (AMTH 491).

REQUIREMENTS OF THE MAJOR

Prerequisites MATH 120 or ENAS 151, and MATH 222 or 225, or equivalents; ENAS 130, CPSC 100 or 112

Number of courses *B.A.* – 11 term courses beyond prereqs (incl senior req); *B.S.* – 14 term courses beyond prereqs (incl senior req)

Specific courses required *B.A.* – ENAS 194 or MATH 246; S&DS 241 or S&DS 238; S&DS 361 or S&DS 230; AMTH 244 or CPSC 202; *B.S.* – same, plus MATH 300 or 301

Distribution of courses *B.A.* – at least 3 courses in a field of concentration concerning the application of math to that field, at least 2 of them advanced; 3 addtl courses as specified; *B.S.* – same, with 2 addtl courses as specified

Substitution permitted MATH 230, 231 for mathematics prerequisites

Senior requirement Senior sem (AMTH 490) or special project (AMTH 491)

FACULTY ASSOCIATED WITH THE PROGRAM OF APPLIED MATHEMATICS

Professors Andrew Barron (*Statistics*), Donald Brown (*Emeritus*) (*Economics, Mathematics*), Joseph Chang (*Statistics*), Ronald Coifman (*Mathematics*), Stanley Eisenstat (*Computer Science*), Michael Fischer (*Computer Science*), Igor Frenkel (*Mathematics*), Roger Howe (*Emeritus*) (*Mathematics*), Peter Jones (*Mathematics*), A. Stephen Morse (*Electrical Engineering*), David Pollard (*Statistics*), Nicholas Read (*Physics, Applied Physics*), Vladimir Rokhlin (*Computer Science, Mathematics*), Peter Schultheiss (*Emeritus*) (*Electrical Engineering*), Martin Schultz (*Emeritus*) (*Computer Science*), Mitchell Smooke (*Mechanical Engineering, Applied Physics*), Daniel Spielman (*Computer Science*), Van Vu (*Mathematics*), Günter Wagner (*Ecology & Evolutionary Biology*), Xiao-Jing Wang (*Neurobiology*), John Wettlaufer (*Geology & Geophysics, Mathematics, Physics*), Huibin Zhou (*Statistics*), Steven Zucker (*Computer Science, Biomedical Engineering*)

Associate Professors John Emerson (*Statistics*), Thierry Emonet (*Molecular, Cellular, & Developmental Biology, Physics*), Josephine Hoh (*Epidemiology & Public Health*), Yuval Kluger (*Pathology*), Michael Krauthammer (*Pathology*), Sekhar Tatikonda (*Electrical Engineering, Statistics*)

J. W. Gibbs Assistant Professors Xiuyuan Cheng, Alexander Cloninger, Manas Rachh, Guy Wolf

Introductory Courses

AMTH 160b / MATH 160b / S&DS 160b, The Structure of Networks Staff

Network structures and network dynamics described through examples and applications ranging from marketing to epidemics and the world climate. Study of social and biological networks as well as networks in the humanities. Mathematical graphs provide a simple common language to describe the variety of networks and their properties. QR

AMTH 222a or b / MATH 222a or b, Linear Algebra with Applications Staff

Matrix representation of linear equations. Gauss elimination. Vector spaces. Linear independence, basis, and dimension. Orthogonality, projection, least squares approximation; orthogonalization and orthogonal bases. Extension to function spaces. Determinants. Eigenvalues and eigenvectors. Diagonalization. Difference equations and matrix differential equations. Symmetric and Hermitian matrices. Orthogonal and unitary transformations; similarity transformations. After MATH 115 or equivalent. May not be taken after MATH 225. QR

Intermediate and Advanced Courses

AMTH 244a / MATH 244a, Discrete Mathematics Ross Berkowitz

Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients, Stirling numbers. Properties of finite set systems. Recommended preparation: MATH 115 or equivalent. QR

AMTH 247b / G&G 247b / MATH 247b, Partial Differential Equations Staff

Introduction to partial differential equations, wave equation, Laplace's equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225, MATH 246, and ENAS 194, or equivalents. QR

AMTH 260a / MATH 260a, Basic Analysis in Function Spaces Ronald Coifman

Diagonalization of linear operators, with applications in physics and engineering; calculus of variations; data analysis. MATH 260 is a natural continuation of PHYS 301. Prerequisites: MATH 120, and 222 or 225. QR

AMTH 262a / CPSC 262a / S&DS 262a, Computational Tools for Data Science Sahand Negahban

An introduction to computational tools for data science. The analysis of data using regression, classification, clustering, principal component analysis, independent component analysis, dictionary learning, topic modeling, dimension reduction, and network analysis. Optimization by gradient methods and alternating minimization. The application of high performance computing and streaming algorithms to the analysis of large data sets. Prerequisites: linear algebra, multivariable calculus, programming. Prerequisites: after or concurrently with MATH 222, 225, or 231; after or concurrently with MATH 120, 230, or ENAS 151; after or concurrently with CPSC 100, 112, or ENAS 130. QR

*** AMTH 342a / EENG 442a, Linear Systems** A. Stephen Morse

Introduction to finite-dimensional, continuous, and discrete-time linear dynamical systems. Exploration of the basic properties and mathematical structure of the linear systems used for modeling dynamical processes in robotics, signal and image processing, economics, statistics, environmental and biomedical engineering, and control theory. Prerequisite: MATH 222 or permission of instructor. QR

AMTH 361b / S&DS 361b, Data Analysis Staff

Selected topics in statistics explored through analysis of data sets using the R statistical computing language. Topics include linear and nonlinear models, maximum likelihood, resampling methods, curve estimation, model selection, classification, and clustering. After S&DS 242 and MATH 222 or 225, or equivalents. QR

AMTH 428a / E&EB 428a / G&G 428a / PHYS 428a, Science of Complex Systems Jun Korenaga

Introduction to the quantitative analysis of systems with many degrees of freedom. Fundamental components in the science of complex systems, including how to simulate complex systems, how to analyze model behaviors, and how to validate models using observations. Topics include cellular automata, bifurcation theory, deterministic chaos, self-organized criticality, renormalization, and inverse theory. Prerequisite: PHYS 301, MATH 247, or equivalent. QR, SC

*** AMTH 437a / ECON 413a / EENG 437a / S&DS 430a, Optimization Techniques** Sekhar Tatikonda

Fundamental theory and algorithms of optimization, emphasizing convex optimization. The geometry of convex sets, basic convex analysis, the principle of optimality, duality. Numerical algorithms: steepest descent, Newton's method, interior point methods, dynamic programming, unimodal search. Applications from engineering and the sciences. Prerequisites: MATH 120 and 222, or equivalents. May not be taken after AMTH 237. QR

AMTH 445a / CPSC 445a, Introduction to Data Mining Guy Wolf

Study of algorithms and systems that allow computers to find patterns and regularities in databases, to perform prediction and forecasting, and to improve their performance generally through interaction with data. After CPSC 202, 223, and MATH 222, or equivalents. QR

*** AMTH 480a or b, Directed Reading** John Wettlaufer

Individual study for qualified students who wish to investigate an area of applied mathematics not covered in regular courses. A student must be sponsored by a faculty member who sets the requirements and meets regularly with the student. Requires a written plan of study approved by the faculty adviser and the director of undergraduate studies.

*** AMTH 482a or b, Research Project** John Wettlaufer

Individual research. Requires a faculty supervisor and the permission of the director of undergraduate studies. The student must submit a written report about the results of the project. May be taken more than once for credit.

*** AMTH 491a or b, Senior Project** John Wettlaufer

Individual research that fulfills the senior requirement. Requires a faculty supervisor and the permission of the director of undergraduate studies. The student must submit a written report about the results of the project.