APPLIED MATHEMATICS

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Mathematical models are widely used throughout science and engineering in fields as diverse as physics, bioinformatics, robotics, image processing, and economics. Despite the broad range of applications, there are a few essential techniques used in addressing most problems. The Applied Mathematics major provides a foundation in these mathematical techniques and trains the student to use them in a substantive field of application.

The interdisciplinary major permits a great deal of flexibility in design. It is intended to appeal to students who wish to study the more mathematical aspects of science or engineering, as well as those whose primary interest is in mathematics and statistics and who wish to become acquainted with applications. Core courses are drawn from Computer Science, Mathematics, Statistics and Data Science, and Engineering and Applied Science. Courses applying mathematics may be drawn from participating programs in Applied Physics; Astronomy; the biological sciences, including Ecology and Evolutionary Biology, Molecular Biophysics and Biochemistry, and Molecular, Cellular, and Developmental Biology; Chemistry; Economics; the various programs in engineering, including Biomedical, Chemical, Electrical, Environmental, and Mechanical Engineering; Geology and Geophysics; Physics; and Political Science. The Applied Mathematics degree program requires a three-course concentration in a field in which mathematics is used.

Students may pursue a major in Applied Mathematics as one of two majors and can thereby equip themselves with mathematical modeling skills while being fully engaged in a field of application. In this case, the concentration requirement of the Applied Mathematics program is flexible in order to recognize the contribution of the other major. A two-course overlap is permitted in satisfying the requirements of the two majors.

PREREQUISITE AND INTRODUCTORY COURSES

Multivariable calculus and linear algebra are required and should be taken before or during the sophomore year. This requirement may be satisfied by MATH 120 or ENAS 151, and MATH 222 or 225 or equivalents. It may also be satisfied by MATH 230, 231. Computer programming skills are also required and may be acquired by taking ENAS 130, CPSC 100 , or 112. Details of individual programs must be worked out in consultation with the director of undergraduate studies, whose signed permission is required.

REQUIREMENTS OF THE MAJOR

The B.A. degree program The program requires eleven term courses beyond the prerequisites, including the senior project, comprising a coherent program:

1. A course in differential equations (ENAS 194 or MATH 246)
2. A course in probability (S&DS 241 or S&DS 238)
3. A course in data analysis (S&DS 361 or S&DS 230)
4. A course in discrete mathematics (AMTH 244 or CPSC 202)
5. Courses in at least three of the following areas including, but not limited to: (a) optimization: AMTH 437; (b) probability and statistics: S&DS 242, S&DS 251, S&DS 312, S&DS 364, ECON 136, ENAS 496; (c) partial differential equations and analysis: MATH 247, 250, 260, 300, 301, 310; (d) algorithms and numerical methods: CPSC 365, 440, ENAS 440, 441; (e) graph theory: AMTH 462; (f) mathematical economics: ECON 350, 351; (g) electrical engineering: EENG 397, 436, 442, S&DS 364; (h) data mining and machine learning: S&DS 365, CPSC 445; (i) biological modeling and computation: CPSC 475, BENG 445, ENAS 391; (j) physical sciences: ASTR 320, 420, G&G 322, 323, 421, PHYS 344, 401, 402, 410, 420, 430, 440, 442, 460, APHY 439, 448; (k) engineering: MENG 280, 285, 361, 383, 463, 469, CENG 301, 315. Because departmental curricula from which the program draws regularly change, the DUS maintains a more exhaustive list of courses satisfying this particular requirement.
6. At least three advanced courses in a field of concentration involving the application of mathematics to that field. Programs in science, engineering, computer science, statistics, and economics are natural sources of concentration. Alternatively, when two majors are undertaken, if the second major is in a participating program, then, recognizing that there can be an overlap of two courses, the student may take for the remaining course an additional choice relevant to the Applied Mathematics major such as listed in point 5 above or for the B.S. degree below. Details of a student's program to satisfy the concentration requirement must be worked out in consultation with, and approved by, the director of undergraduate studies.

The B.S. degree program In addition to the courses indicated for the B.A. degree, the B.S. degree, which totals fourteen term courses beyond the prerequisites, must also include:

1. Topics in analysis (MATH 300) or introduction to analysis (MATH 301); the course selected may not be counted toward the area requirement for the major (see item 5 above)
2. An additional course selected from the list in item 5 above

3. Another course numbered 300 or higher from the list above, or a course numbered 300 or higher in mathematics, applied mathematics, statistics, or quantitative computer science or engineering, subject to the approval of the director of undergraduate studies

Alternatively, students may petition to receive a B.S. in Applied Mathematics by fulfilling the B.A. requirements in Applied Mathematics and the B.S. requirements in another program.

Credit/D/Fail A maximum of one course credit taken Credit/D/Fail may be counted toward the requirements of the major.

SENIOR REQUIREMENT

Both the B.A. and B.S. degree programs require Senior Seminar and Project (AMTH 490), or a special project completed during senior year (AMTH 491).

REQUIREMENTS OF THE MAJOR

Prerequisites MATH 120 or ENAS 151, and MATH 222 or 225, or equivalents; ENAS 130, CPSC 100 or 112
Number of courses *B.A.*—11 term courses beyond prereqs (incl senior req); *B.S.*—14 term courses beyond prereqs (incl senior req)
Specific courses required *B.A.*—ENAS 194 or MATH 246; S&DS 241 or S&DS 238; S&DS 361 or S&DS 230; AMTH 244 or CPSC 202; *B.S.*—same, plus MATH 300 or 301
Distribution of courses *B.A.*—at least 3 advanced courses in a field of concentration concerning the application of math to that field; 3 addtl courses as specified; *B.S.*—same, with 2 addtl courses as specified
Substitution permitted MATH 230, 231 for mathematics prerequisites
Senior requirement Senior sem (AMTH 490) or special project (AMTH 491)

FACULTY ASSOCIATED WITH THE PROGRAM OF APPLIED MATHEMATICS

Professors Andrew Barron (*Statistics*), Donald Brown (*Emeritus*) (*Economics, Mathematics*), Joseph Chang (*Statistics),* Ronald Coifman *(Mathematics*), Stanley Eisenstat (*Computer Science*), Michael Fischer (*Computer Science*), Igor Frenkel (*Mathematics*), Roger Howe (*Emeritus)* (*Mathematics*), Peter Jones (*Mathematics*), A. Stephen Morse (*Electrical Engineering*), David Pollard (*Statistics*), Nicholas Read (*Physics, Applied Physics*), Vladimir Rokhlin (*Computer Science, Mathematics*), Peter Schultheiss (*Emeritus*) (*Electrical Engineering*), Martin Schultz (*Emeritus*) (*Computer Science*), Mitchell Smooke (*Mechanical Engineering, Applied Physics*), Daniel Spielman (*Computer Science*), Mary-Louise Timmermans (Geology & Geophysics), Van Vu (*Mathematics*), Günter Wagner (*Ecology & Evolutionary Biology*), Xiao-Jing Wang (*Neurobiology*), John Wettlaufer (*Geology & Geophysics, Mathematics, Physics*), Huibin Zhou (*Statistics*), Steven Zucker (*Computer Science, Biomedical Engineering*)

Associate Professors John Emerson (*Statistics*), Thierry Emonet (*Molecular, Cellular, & Developmental Biology, Physics*), Josephine Hoh (*Epidemiology & Public Health*), Yuval Kluger (*Pathology*), Michael Krauthammer (*Pathology*), Sekhar Tatikonda (*Electrical Engineering, Statistics*)

J. W. Gibbs Assistant Professors Xiuyuan Cheng, Alexander Cloninger, Manas Rachh, Guy Wolf

Introductory Courses

AMTH 160b / MATH 160b / S&DS 160b, The Structure of Networks Staff
Network structures and network dynamics described through examples and applications ranging from marketing to epidemics and the world climate. Study of social and biological networks as well as networks in the humanities. Mathematical graphs provide a simple common language to describe the variety of networks and their properties. QR

AMTH 222a or b / MATH 222a or b, Linear Algebra with Applications Staff
Matrix representation of linear equations. Gauss elimination. Vector spaces. Linear independence, basis, and dimension. Orthogonality, projection, least squares approximation; orthogonalization and orthogonal bases. Extension to function spaces. Determinants. Eigenvalues and eigenvectors. Diagonalization. Difference equations and matrix differential equations. Symmetric and Hermitian matrices. Orthogonal and unitary transformations; similarity transformations. After MATH 115 or equivalent. May not be taken after MATH 225. QR

Intermediate and Advanced Courses

AMTH 244a / MATH 244a, Discrete Mathematics Ross Berkowitz
Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients, Stirling numbers. Properties of finite set systems. Recommended preparation: MATH 115 or equivalent. QR

AMTH 247b / G&G 247b / MATH 247b, Partial Differential Equations Jeremy Hoskins
Introduction to partial differential equations, wave equation, Laplace's equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225, MATH 246, and ENAS 194, or equivalents. QR

AMTH 260a / MATH 260a, Basic Analysis in Function Spaces Staff
Diagonalization of linear operators, with applications in physics and engineering; calculus of variations; data analysis. MATH 260 is a natural continuation of PHYS 301. Prerequisites: MATH 120, and 222 or 225. QR

AMTH 262a / CPSC 262a / S&DS 262a, Computational Tools for Data Science Staff
An introduction to computational tools for data science. The analysis of data using regression, classification, clustering, principal component analysis, independent component analysis, dictionary learning, topic modeling, dimension reduction, and network analysis. Optimization by gradient methods and alternating minimization. The application of high performance computing and streaming algorithms to the analysis of large data sets. Prerequisites: linear algebra, multivariable calculus, programming. Prerequisites: after or concurrently with MATH 222, 225, or 231; after or concurrently with MATH 120, 230, or ENAS 151; after or concurrently with CPSC 100, 112, or ENAS 130. QR

* **AMTH 342a / EENG 442a, Linear Systems** A. Stephen Morse
Introduction to finite-dimensional, continuous, and discrete-time linear dynamical systems. Exploration of the basic properties and mathematical structure of the linear systems used for modeling dynamical processes in robotics, signal and image processing, economics, statistics, environmental and biomedical engineering, and control theory. Prerequisite: MATH 222 or permission of instructor. QR

AMTH 361b / S&DS 361b, Data Analysis Staff
Selected topics in statistics explored through analysis of data sets using the R statistical computing language. Topics include linear and nonlinear models, maximum likelihood, resampling methods, curve estimation, model selection, classification, and clustering. After S&DS 242 and MATH 222 or 225, or equivalents. QR

AMTH 364b / EENG 454b / S&DS 364b, Information Theory Andrew Barron
Foundations of information theory in communications, statistical inference, statistical mechanics, probability, and algorithmic complexity. Quantities of information and their properties: entropy, conditional entropy, divergence, redundancy, mutual information, channel capacity. Basic theorems of data compression, data summarization, and channel coding. Applications in statistics and finance. After STAT 241. QR

AMTH 428a / E&EB 428a / G&G 428a / PHYS 428a, Science of Complex Systems Jun Korenaga
Introduction to the quantitative analysis of systems with many degrees of freedom. Fundamental components in the science of complex systems, including how to simulate complex systems, how to analyze model behaviors, and how to validate models using observations. Topics include cellular automata, bifurcation theory, deterministic chaos, self-organized criticality, renormalization, and inverse theory. Prerequisite: PHYS 301, MATH 247, or equivalent. QR, SC

* **AMTH 437a / ECON 413a / EENG 437a / S&DS 430a, Optimization Techniques** Sekhar Tatikonda
Fundamental theory and algorithms of optimization, emphasizing convex optimization. The geometry of convex sets, basic convex analysis, the principle of optimality, duality. Numerical algorithms: steepest descent, Newton's method, interior point methods, dynamic programming, unimodal search. Applications from engineering and the sciences. Prerequisites: MATH 120 and 222, or equivalents. May not be taken after AMTH 237. QR

* **AMTH 480a or b, Directed Reading** John Wettlaufer
Individual study for qualified students who wish to investigate an area of applied mathematics not covered in regular courses. A student must be sponsored by a faculty member who sets the requirements and meets regularly with the student. Requires a written plan of study approved by the faculty adviser and the director of undergraduate studies.

* **AMTH 482a or b, Research Project** John Wettlaufer
Individual research. Requires a faculty supervisor and the permission of the director of undergraduate studies. The student must submit a written report about the results of the project. May be taken more than once for credit.

* **AMTH 490b, Senior Seminar and Project** John Wettlaufer
Under the supervision of a member of the faculty, each student works on an independent project. Students participate in seminar meetings at which they speak on the progress of their projects. Some meetings may be devoted to talks by visiting faculty members or applied mathematicians.

* **AMTH 491a or b, Senior Project** John Wettlaufer
Individual research that fulfills the senior requirement. Requires a faculty supervisor and the permission of the director of undergraduate studies. The student must submit a written report about the results of the project.