MATHEMATICS

See also Applied Mathematics

Director of undergraduate studies: Andrew Neitzke, (andrew.neitzke@yale.edu) DL 425; associate director of undergraduate studies: Miki Havlickova (miki.havlickova), DL 446; Math DUS website, Math department website

Mathematics has many aspects: it is the language and tool of the sciences, a cultural phenomenon with a rich historical tradition, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics major provides a broad education in various areas of mathematics in a program flexible enough to accommodate many ranges of interest.

PREREQUISITE

The prerequisite for both the B.A and B.S. degree programs is single variable calculus, through the level of MATH 115 or equivalent.

CALCULUS PLACEMENT PROCEDURES

The department offers a three-term sequence in calculus, MATH 112, 115, and 120. Students who have not taken calculus at Yale and who wish to enroll in calculus must take the mathematics online placement examination. Detailed information is available on the Math first year student resources site. A calculus advising session will be held prior to registration, to answer student questions about placement. MATH 112 covers differential calculus, and assumes mastery of high school algebra, geometry, and trigonometry. Enrolling students are expected to know the basic definitions of the trigonometric functions, inverse functions, factoring quadratic polynomials, and elementary area and volume formulas of plane and solid geometry. Students who could benefit from a review of precalculus are encouraged to consider MATH 110 and 111 in place of MATH 112.

The next course in the calculus sequence is MATH 115, which covers integral calculus, including sequences and series. It assumes mastery of the content of MATH 112 or equivalent (for example, AP Calculus AB).

MATH 120 covers multivariable calculus, and assumes mastery of the material in MATH 115 or equivalent (for example, AP Calculus BC).

INTRODUCTORY SEQUENCE FOR THE MATHEMATICS MAJOR

Students wishing to pursue study of mathematics typically enroll in MATH 225 (linear algebra and introduction to proofs), and MATH 255 (real analysis and introduction to proofs). MATH 225 and MATH 255 can be taken in either order, though it is recommended to take MATH 225 first.

Most students complete multivariable calculus before enrolling in MATH 225, however, prospective mathematics majors and students with interest in abstract mathematics may consider enrolling in MATH 225 directly after MATH 115 or equivalent, and complete their vector analysis/multivariable calculus requirement with MATH 302.

Students with a strong mathematical background that includes exposure to mathematical proofs are encouraged to consider the intensive version of the introductory sequence, MATH 226 and MATH 256.

Incoming students are encouraged to visit the Math first year student resources website for advice about choosing their mathematics courses.

REQUIREMENTS OF THE MAJOR

B.A. degree program The B.A. degree program normally consists of ten term courses in Mathematics numbered 222 or higher, including the senior requirement (MATH 475 or 480 or MATH 481); excluding, however, MATH 470. To acquire both depth and breadth in the field, students are required to take at least two term courses in each of three of the following five categories: analysis; algebra and number theory; statistics and applied mathematics; geometry and topology; and logic and foundations.

Introductory sequence requirement for students in the Class of 2025 and beyond: Each student is expected to complete Linear algebra (MATH 225 or 226), Real analysis (MATH 255 or MATH 356), and Vector analysis or Multivariable calculus (MATH 302 or 120). MATH 222 is not recommended as a substitute for MATH 225 or 226, as it does not provide an introduction to proof writing, which is an essential skill for completing upper level mathematics courses.

Students in the Class of 2022, 2023, and 2024 who have not yet completed their introductory requirement (MATH 230 and 231, or 120 and 225 and 250) are encouraged to visit the Math curriculum revision website for detailed information about transitioning to the new introductory sequences.

B.S. degree program A candidate for the B.S. degree must take at least two advanced term courses in the physical sciences, such as ASTR 418, 420, 430, CHEM 333, 470, or PHYS 401, 402, 410, 412, 420, 430, 440, 441 in addition to the ten term courses required for the B.A. Such courses require the approval of the director of undergraduate studies (DUS); written approval is advised.

Both B.A. and B.S. degree programs Each major program must also include at least one course in at least two of the three core areas: real analysis; algebra; and complex analysis. Taking courses from all three core areas is strongly recommended.
Distinction in the major To be eligible for Distinction in the Major, a student must have completed at least one course from each of the three core areas. The categories and core areas to which each course belongs are indicated in the course listings.

The intensive major Candidates for a degree with an intensive major in Mathematics must take courses in all three of the core areas: real analysis; algebra; and complex analysis. Intensive majors are also expected to include at least two graduate term courses in the Mathematics department, or equivalent independent study, in their programs. Familiarity with the material of the following courses is prerequisite to graduate courses in each category: algebra: MATH 350 and MATH 370; analysis: MATH 305, 310; algebraic topology: MATH 350, 430; logic and foundations: MATH 270.

Substitutions With permission of the Math DUS, up to two courses from other departments may be counted towards the required courses. For a list of courses that are typically approved, visit the FAQ page on the Math department website.

Credit/D/Fail Courses taken Credit/D/Fail may not be counted toward the requirements of the major.

Roadmap See visual roadmap of the requirements.

SENIOR REQUIREMENT
During the senior year students majoring in Mathematics normally take the senior seminar (MATH 480 or MATH 481). Alternatively, with the consent of the DUS, students may write a senior essay in MATH 475 under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the DUS at least six months prior to enrolling in MATH 475, and are encouraged to pursue independent study opportunities prior to their senior year, for example through the Mathematics directed reading program or through summer research programs.

ADVISING
Students interested in pursuing further study in pure mathematics should include MATH 302, 305, 310, 350, 370, and 430 in their programs, and should consider taking one or more graduate-level courses. Students interested in applications of mathematics should include MATH 302, 310, 350, and a selection of courses from MATH 241, 242, 244, 246, 251, 260, and CPSC 440.

Courses related to mathematics Each Mathematics major is urged to acquire additional familiarity with the uses of mathematics by taking courses in Applied Mathematics, Computer Science, Engineering and Applied Science, Economics, Philosophy, Physics, Statistics & Data Science, or other departments. In some instances a limited number of such courses may be counted among the ten courses required for the major in Mathematics, with the approval of the DUS.

Graduate work Each year the Mathematics and Statistics & Data Science departments offer a large number of graduate courses, some of which are accessible to undergraduates with advanced preparation in mathematics. Further information may be obtained from the DUSes, whose permission, with that of the relevant director of graduate studies, is required for admission.

Combined B.S./M.S. degree program Students who, by the end of their senior year, complete the requirements of the department for the M.S. in Mathematics are eligible to receive this degree at their Senior Commencement. Required are: (1) eight term courses numbered 500 or higher, most of which must be completed with grades of B or better; (2) passing a written qualifying examination of the student’s choice from analysis, algebra, or topology.

The master’s program is in no sense a substitute for the B.A. or B.S. program; rather, it is designed to accommodate exceptional students who, by means of accelerated or independent study, can satisfy the department as to their command of the content of the normal undergraduate program by the end of the junior year. Candidates must contact the Mathematics DUS at least two weeks prior to the last day of classes of their fifth term at Yale College. Minimum eligibility criteria include at least seventy-five percent of A/A– grades within mathematics as well as seventy-five percent of A/A– grades overall. For more information on mathematics requirements, please see the B.S./M.S. section of the Math major FAQ. For more information on Yale College requirements for the program, see Section K, Special Arrangements, “Simultaneous Award of the Bachelor’s and Master’s Degrees,” in the Academic Regulations.

REQUIREMENTS OF THE MAJOR
Prerequisite Single-variable calculus through MATH 115 or equivalent

Number of courses B.A. – 10 term courses numbered 222 or higher (incl senior essay), excludes MATH 470; B.S. – same, with 2 addtl adv courses in physical sciences approved by DUS

Specific courses required B.A. and B.S. – MATH 225 or MATH 226; MATH 235 or MATH 256; MATH 302 or MATH 120

Distribution of courses B.A. and B.S. – 2 courses in each of 3 categories chosen from: analysis; algebra and number theory; stat and applied math; geometry and topology; logic and foundations; 1 course from 2 of 3 core areas chosen from: real analysis; algebra; and complex analysis

Substitution permitted With DUS permission, up to 2 courses from other depts, as specified

Intensive major Courses in all 3 core areas; 2 MATH grad courses or equivalent independent study counted among the required courses

Senior requirement Senior sem (MATH 480 or MATH 481) or, with DUS permission, senior essay (MATH 475) and oral report

Mathematics has many aspects. It is the language and tool of the sciences, a key part of cultural development since ancient times, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics program provides a broad education in various areas of mathematics and is flexible enough to accommodate many interests. Mathematics majors have numerous options after graduation, including graduate study in mathematics or in various fields of application, government or
corporate laboratory work, consulting, finance and banking, and teaching. Students considering a major in Mathematics are encouraged to consult with the director of undergraduate studies (DUS) during their first year.

A variety of resources are available to students who want additional help in any mathematics course:

- Each course instructor holds office hours for students multiple times a week.
- Undergraduate peer tutors and learning assistants associated with many of the courses provide help through regular office hours.
- The Mathematics department offers coaching sessions for students in calculus courses.
- Drop-in tutoring with Math & Science Tutors is available in most residential colleges and at the Poorvu Center for Teaching and Learning. For details, see the Poorvu Center STEM Tutoring & Programs web page.

COURSES

A full description of offered mathematics courses can be found in Yale Course Search. The department offers several courses that satisfy the QR distributional requirement and do not assume knowledge of mathematics beyond the basic high-school level. They include:

- MATH 106, The Shape of Space
- MATH 107, Mathematics in the Real World
- MATH 108, Estimation and Error

The courses below include the ones most frequently taken by first-year students. Enrollment requires appropriate placement based on the online placement exam.

- MATH 110 (fall) and MATH 111 (spring) form a two-term introduction to differentiation, in which precalculus and calculus topics are combined. Successful completion of MATH 110 and 111 is equivalent to MATH 112 in that it satisfies the same major and professional-school (e.g. premed) requirements; however, a student completing MATH 110 and 111 receives two course credits and two QR credits. Only students who complete MATH 110 will be allowed to enroll in MATH 111. No prior exposure is assumed; some knowledge of algebra and precalculus mathematics is helpful.
- MATH 112 is the introductory course for students with a strong foundation in high school mathematics. It focuses primarily on differentiation and assumes no previous exposure to calculus.
- MATH 115 builds on MATH 112 and focuses primarily on integration and infinite series. Knowledge of topics covered in MATH 112 is assumed.
- MATH 120 combines the ideas and techniques of one-variable calculus with vector geometry and algebra to deal with geometrical situations in planes and in three-dimensional space. Knowledge of topics covered in MATH 112 and MATH 115 or 116 is assumed.

The courses below serve as options after MATH 112 for calculus students who have a specific focus. Students interested in the biological sciences should consider MATH 116 (fall) and/or MATH 121 (spring).

- MATH 116 is designed for bioscience and premedical students. It combines differential equations with modeling and applications in biology. Knowledge of topics covered in MATH 112 is assumed.
- MATH 118 is designed for students interested in economics and social sciences who do not intend to take additional mathematics courses. It covers basic ideas in linear algebra as well as differential calculus of several variables. Knowledge of topics covered in MATH 112 is assumed, MATH 115 is recommended.
- MATH 121 is designed for bioscience and premedical students. It combines multivariable calculus with applications in biology. Knowledge of topics covered in MATH 115 or MATH 116 is assumed.

The higher-level courses below assume at least a year of calculus and are available to well prepared first-year students.

- MATH 222, MATH 225 and MATH 226 deal with linear algebra, the common language for a wide variety of applications involving many variables.
- MATH 222 emphasizes computations and applications of linear algebra, and is especially useful for students who wish to major in engineering, social sciences, economics, and other fields. Students normally enroll in MATH 222 after completing MATH 120, but well prepared students may consider taking MATH 222 after MATH 115 or equivalent.
- MATH 225 and MATH 226 are proof-based courses focusing on geometric and conceptual issues and the logical structure of the subject. These are recommended for students who wish to major in mathematics and for students who intend to take higher-level mathematics courses. MATH 225 is typically taken by students who have completed multivariable calculus, or students who have completed integral calculus and have strong interest in proof-based mathematics courses. MATH 225 assumes no prior exposure to writing mathematical proofs. MATH 226 is an intensive version of MATH 225 for students with strong mathematical background who have some familiarity with writing mathematical proofs.
- MATH 255 and MATH 256 are proof-based courses that serve as introduction to rigorous analysis on the line with some applications. It assembles the conceptual tools that underline the workings of calculus and prepares the student for further work in analysis. Emphasis is on reading, discovering, and writing clear proofs. MATH 255 and MATH 256 are accessible after integral calculus, though

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most students will first have completed MATH 225 or MATH 226. MATH 255 will begin with introduction to writing mathematical proofs. MATH 256 is an intensive version of MATH 255, and assumes prior exposure to writing mathematical proofs.

Additional higher-level courses may be open to exceptionally well prepared first-year students. Interested students should seek advice from the DUS.

FACULTY OF THE DEPARTMENT OF MATHEMATICS

Professors Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Anna Gilbert, Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Richard Kenyon, Ivan Losev, Gregory Margulis, Yair Minsky, Vincent Moncrief, Andrew Neitzke, Hee Oh, †Nicholas Read, Vladimir Rokhlin, Wilhelm Schlag, George Seligman (Emeritus), †Daniel Spielman, Van Vu, Lu Wang, †John S. Wettlaufer, Gregg Zuckerman (Emeritus)

J. W. Gibbs Assistant Professors Yariv Aizenbud, Pablo Boixeda Alvarez, Subhadip Dey, Gurbir Dhillon, Daniel Douglas, James Farre, Abinand Gopal, Erik Orvehed Hiltunen, YakovKononov, Boris Landa, Or Landesberg, Kevin O’Neill, Franco Vargas Pallete, Cosmin Pohoata, Congling Qiu, Ebru Toprak.

Adjunct Professors Gil Kalai, Alex Lubotzky, Jacques Peyriere, Mathias Schacht

Senior Lecturers John Hall, Miki Havlickova.

Lecturers Ian Adelstein, Mihai Alboiu, James Barnes, Rachel Diethorn, Eric Geiger, Su Ji Hong, Robert McDonald, Brett Smith.

†A joint appointment with primary affiliation in another department.

View Courses