MATHEMATICS

See also Applied Mathematics.

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Mathematics has many aspects: it is the language and tool of the sciences, a cultural phenomenon with a rich historical tradition, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics major provides a broad education in various areas of mathematics in a program flexible enough to accommodate many ranges of interest.

PREREQUISITE

The prerequisite for both the B.A and B.S. degree programs is calculus through the level of MATH 120 or the equivalent.

PLACEMENT PROCEDURES

The department offers a three-term sequence in calculus, MATH 112, 115, and 120. Students who have not taken calculus at Yale and who wish to enroll in calculus must take the mathematics online placement examination; a link to the online examination and additional information are available on the departmental website. A calculus advising session will be held at the beginning of the fall term to answer student questions about placement.

MATH 112 is an introductory course that presupposes basic skills in high school algebra, geometry, and trigonometry. Enrolling students are expected to know the basic definitions of the trigonometric functions, synthetic division, factorization, and elementary area and volume formulas of plane and solid geometry. MATH 115 presupposes familiarity with the topics covered in MATH 112. MATH 120 presupposes familiarity with the topics covered in MATH 115.

MATH 230, 231 is an advanced course sequence in linear algebra and introductory analysis for students with exceptionally strong backgrounds in mathematics. Students who wish to enroll in MATH 230 should consult with the instructor of the course. After MATH 115, students with a strong interest in abstract mathematics should consider taking MATH 230, 231.

REQUIREMENTS OF THE MAJOR

B.A. degree program The B.A. degree program normally consists of ten term courses in Mathematics numbered 222 or higher, including MATH 480. These ten may include no more than five term courses from other institutions. Each student is expected to take vector calculus and linear algebra: either MATH 230 and 231, or either one of MATH 222 or 225 with MATH 250. To acquire both depth and breadth in the field, students are required to take at least two term courses in each of three of the following five categories: analysis, algebra and number theory, statistics and applied mathematics, geometry and topology, and logic and foundations.

B.S. degree program A candidate for the B.S. degree must take at least two advanced term courses in the physical sciences, such as CHEM 328, 332, 333, or PHYS 401, 402, in addition to the ten term courses required for the B.A. degree. Such courses require the approval of the DUS; written approval is advised.

Both B.A. and B.S. degree programs Each major program must also include at least one course in at least two of the three core areas: real analysis, algebra, and complex analysis. Taking courses from all three core areas is strongly recommended.

Distinction in the major To be eligible for Distinction in the Major, a student must have completed at least one course from each of the three core areas. The categories and core areas to which each course belongs are indicated in the course listings.

The intensive major Candidates for a degree with an intensive major in Mathematics must take courses in all three of the core areas: real analysis, algebra, and complex analysis. Intensive majors are also expected to include at least two graduate term courses in the Mathematics department, or equivalent independent study, in their programs. Familiarity with the material of the following courses is prerequisite to graduate courses in each category: *algebra:* two courses between 350 and 399; *analysis:* MATH 301, 305, 310; *algebraic topology:* MATH 301, 350; *logic and foundations:* MATH 270.

Credit/D/Fail Courses taken Credit/D/Fail may not be counted toward the requirements of the major.

SENIOR REQUIREMENT

During the senior year students majoring in Mathematics normally take the senior seminar (MATH 480). Alternatively, with the consent of the DUS, highly qualified students may write a senior essay in MATH 475 under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the DUS early in the fall term.

ADVISING

Any student interested in pursuing further study in pure mathematics should include MATH 301, 305, 310, 350, 370, and 430 in his or her program, and should consider taking one or more graduate-level courses. Students interested in applications of mathematics should include MATH 300 or 301, 310, 350, and a selection of courses from MATH 241, 242, 244, 246, 251, 260, and CPSC 440.

Courses related to mathematics Each Mathematics major is urged to acquire additional familiarity with the uses of mathematics by taking courses in Applied Mathematics, Computer Science, Engineering and Applied Science, Economics, Philosophy,

Physics, Statistics & Data Science, or other departments. In some instances a limited number of such courses may be counted among the ten courses required for the major in Mathematics, with the approval of the DUS.

Graduate work Each year the Mathematics and Statistics & Data Science departments offer a large number of graduate courses, some of which are accessible to undergraduates with advanced preparation in mathematics. Further information may be obtained from the directors of undergraduate studies whose permission, with that of the relevant director of graduate studies, is required for admission.

Combined B.S./M.S. degree program Students who, by the end of their senior year, complete the requirements of the department for the M.S. in Mathematics are eligible to receive this degree at their Senior Commencement. Required are: (1) eight term courses numbered 500 or higher, most of which must be completed with grades of B or better; (2) satisfactory performance on a general oral examination.

The master's program is in no sense a substitute for the B.A. or B.S. program; rather, it is designed to accommodate a very few exceptional students who, by means of accelerated or independent study, can satisfy the department as to their command of the content of the normal undergraduate program by the end of the junior year. Candidates must submit a proposal that foresees this level of achievement to the DUS no later than the last day of classes in their fifth term of enrollment in Yale College. If approved by the department, the proposal is forwarded to the Yale College Dean's Office. Students' status and progress are reviewed before they are permitted to continue in the program in the senior year. For more information on Yale College requirements for the program, see "Simultaneous Award of the Bachelor's and Master's Degrees" in Section K, Special Arrangements, in the Academic Regulations.

Students take at least two graduate term courses in the junior year (normally courses in algebra or analysis are the first graduate courses taken). The general oral examination covers a list of topics available from the director of graduate studies and is accepted in lieu of the usual senior oral presentation. Details concerning the requirements for the master's degree may be obtained from the director of graduate studies.

REQUIREMENTS OF THE MAJOR

Prerequisite MATH 120 or equivalent
Number of courses *B.A.*—10 term courses numbered 222 or higher, incl MATH 480; *B.S.*—same, with 2 addtl adv courses in physical sciences approved by DUS
Specific courses required MATH 230 and 231; or MATH 222 or 225 with MATH 250
Distribution of courses *B.A. and B.S.*—2 courses in each of 3 categories chosen from analysis, algebra and number theory, stat and applied math, geometry and topology, logic and foundations; courses from 2 of 3 core areas, as specified
Substitution permitted With DUS permission, certain courses in Applied Math, Comp Sci, Engineering & Applied Science, Econ, Phil, Physics, S&DS, or other depts
Intensive major Courses in all 3 core areas; 2 MATH grad courses or equivalent independent study counted among the required courses
Senior requirement Senior sem (MATH 480) or, with DUS permission, senior essay (MATH 475) and oral report

FACULTY OF THE DEPARTMENT OF MATHEMATICS

Professors Jeffrey Brock, Andrew Casson (*Emeritus*), Ronald Coifman, Igor Frenkel, Alexander Goncharov, Peter Jones, Gregory Margulis, Yair Minsky, Vincent Moncrief, Hee Oh, Sam Payne, *Nicholas Read, Vladimir Rokhlin, Wilhelm Schlag, *Daniel Spielman, Van Vu, *John S. Wettlaufer, Gregg Zuckerman

Assistant Professor Stefan Steinerberger

J. W. Gibbs Assistant Professors Asher Auel, Ross Berkowitz, Pat Devlin, Jeremy Hoskins, Max Kutler, Arie Levit, Yuchen Liu, Kalina Mincheva, Gal Mishne, Fei Qi, Kirill Serkh, Gil Shabat, Oleksandr Tsymbaliuk, Caglar Uyanik, Tom VandenBoom, Anibal Velozo, Philsang Yoo

Adjunct Professors Gil Kalai, Alex Lubotzky

Senior Lecturer Marketa Havlickova

Lecturers Ian Adelstein, John Hall, Sudesh Kalyanswamy, Itziar Ochoa de Alaiza Gracia, Pam Sargent, Brett Smith, Sarah Vigliotta

Statistics Faculty Andrew Barron, Joseph Chang, John Hartigan (*Emeritus*), David Pollard

* A joint appointment with primary affiliation in another department.

Courses

*** MATH 107a, Mathematics in the Real World** Erik Rosenthal
The use of mathematics to address real-world problems. Applications of exponential functions to compound interest and population growth; geometric series in mortgage payments, amortization of loans, present value of money, and drug doses and blood levels; basic probability, Bayes's rule, and false positives in drug testing; elements of logic. No knowledge of calculus required. Enrollment limited to students who have not previously taken a high school or college calculus course. QR

MATH 108b, Estimation and Error Pamela Sargent
A problem-based investigation of basic mathematical principles and techniques that help make sense of the world. Estimation, order of magnitude, approximation and error, counting, units, scaling, measurement, variation, simple modeling. Applications to demographics,

geology, ecology, finance, and other fields. Emphasis on both the practical and the philosophical implications of the mathematics. Does not count toward the requirements of a major in Mathematics. QR

* **MATH 110a, Introduction to Functions and Calculus I** Staff
Comprehensive review of precalculus, limits, differentiation and the evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of MATH 110 and 111 is equivalent to MATH 112. No prior acquaintance with calculus is assumed; some knowledge of algebra and precalculus mathematics is helpful. QR

* **MATH 111b, Introduction to Functions and Calculus II** Staff
Continuation of MATH 110. Comprehensive review of precalculus, limits, differentiation and evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of both MATH 110 and 111 is equivalent to MATH 112. Prerequisite: MATH 110. QR

* **MATH 112a or b, Calculus of Functions of One Variable I** Staff
Limits and their properties. Definitions and some techniques of differentiation and the evaluation of definite integrals, with applications. Use of the software package Mathematica to illustrate concepts. No prior acquaintance with calculus or computing assumed. May not be taken after MATH 110 or 111. QR

* **MATH 115a or b, Calculus of Functions of One Variable II** Staff
A continuation of MATH 112. Applications of integration, with some formal techniques and numerical methods. Improper integrals, approximation of functions by polynomials, infinite series. Exercises involve the software package Mathematica. After MATH 112 or equivalent; open to freshmen with some preparation in calculus. May not be taken after MATH 116. QR

* **MATH 116a or b, Mathematical Models in the Biosciences I: Calculus Techniques** John Hall
Introduction to topics in mathematical modeling that are applicable to biological systems. Discrete and continuous models of population, neural, and cardiac dynamics. Stability of fixed points and limit cycles of differential equations. Applications include Norton's chemotherapy scheduling and stochastic models of tumor suppressor gene networks. After MATH 112 or equivalent. May not be taken after MATH 115. QR

* **MATH 118a or b, Introduction to Functions of Several Variables** Staff
A combination of linear algebra and differential calculus of several variables. Matrix representation of linear equations, Gauss elimination, vector spaces, independence, basis and dimension, projections, least squares approximation, and orthogonality. Three-dimensional geometry, functions of two and three variables, level curves and surfaces, partial derivatives, maxima and minima, and optimization. Intended for students in the social sciences, especially Economics. May not be taken after MATH 120 or 222. Prerequisite: MATH 112. QR

* **MATH 120a or b, Calculus of Functions of Several Variables** Staff
Analytic geometry in three dimensions, using vectors. Real-valued functions of two and three variables, partial derivatives, gradient and directional derivatives, level curves and surfaces, maxima and minima. Parametrized curves in space, motion in space, line integrals; applications. Multiple integrals, with applications. Divergence and curl. The theorems of Green, Stokes, and Gauss. After MATH 115, or with permission of instructor. May not be taken after MATH 121. QR

MATH 160b / AMTH 160b / S&DS 160b, The Structure of Networks Ariel Jaffe
Network structures and network dynamics described through examples and applications ranging from marketing to epidemics and the world climate. Study of social and biological networks as well as networks in the humanities. Mathematical graphs provide a simple common language to describe the variety of networks and their properties. QR

* **MATH 199a, Mathematical Problem Solving** Patrick Devlin
Study of a variety of techniques for solving challenging mathematical problems. Topics include the pigeon-hole principle, probabilities, congruences, generating functions, polynomials, and basic number theory. Open to students with beginning, intermediate, or advanced problem-solving skills. This course may be taken twice for credit and is graded pass/fail. ½ Course cr

MATH 222a or b / AMTH 222a or b, Linear Algebra with Applications Staff
Matrix representation of linear equations. Gauss elimination. Vector spaces. Linear independence, basis, and dimension. Orthogonality, projection, least squares approximation; orthogonalization and orthogonal bases. Extension to function spaces. Determinants. Eigenvalues and eigenvectors. Diagonalization. Difference equations and matrix differential equations. Symmetric and Hermitian matrices. Orthogonal and unitary transformations; similarity transformations. After MATH 115 or equivalent. May not be taken after MATH 225. QR

MATH 225a or b, Linear Algebra and Matrix Theory Staff
An introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, and quadratic forms. Some relations to calculus and geometry are included. After or concurrently with MATH 120. May not be taken after MATH 222. QR

* **MATH 230a, Vector Calculus and Linear Algebra I** Patrick Devlin
A careful study of the calculus of functions of several variables, combined with linear algebra. QR

* **MATH 231b, Vector Calculus and Linear Algebra II** Patrick Devlin
Continuation of MATH 230. Application of linear algebra to differential calculus. Inverse and implicit function theorems; the idea of a manifold; integration of differential forms; general Stokes' theorem. QR

* **MATH 235b, Reflection Groups** Igor Frenkel
Concepts of linear algebra are used to explore the algebraic and geometric properties of groups generated by reflections. Examples from reflection groups introduce elements of group theory, Lie algebras, and representation theory. Reflections in a real Euclidean space, groups generated by reflections, crystallographic groups, and Coxeter groups. Preference to sophomores majoring in mathematics or the sciences. Prerequisite: MATH 222 or 225. QR

MATH 241a / S&DS 241a, Probability Theory Yihong Wu
Introduction to probability theory. Topics include probability spaces, random variables, expectations and probabilities, conditional probability, independence, discrete and continuous distributions, central limit theorem, Markov chains, and probabilistic modeling. After or concurrently with MATH 120 or equivalent. QR

MATH 242b / S&DS 242b, Theory of Statistics Andrew Barron
Study of the principles of statistical analysis. Topics include maximum likelihood, sampling distributions, estimation, confidence intervals, tests of significance, regression, analysis of variance, and the method of least squares. Some statistical computing. After S&DS 241 and concurrently with or after MATH 222 or 225, or equivalents. QR

MATH 244a or b / AMTH 244a or b, Discrete Mathematics Ross Berkowitz
Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients, Stirling numbers. Properties of finite set systems. Recommended preparation: MATH 115 or equivalent. QR

MATH 246a or b, Ordinary Differential Equations Staff
First-order equations, second-order equations, linear systems with constant coefficients. Numerical solution methods. Geometric and algebraic properties of differential equations. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or equivalent. QR

MATH 247b / AMTH 247b / G&G 247b, Partial Differential Equations Kirill Serkh
Introduction to partial differential equations, wave equation, Laplace's equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225, MATH 246, and ENAS 194, or equivalents. QR

MATH 250a or b, Vector Analysis Staff
Calculus of functions of several variables, using vector and matrix methods. The derivative as a linear mapping. Inverse and implicit function theorems. Transformation of multiple integrals. Line and surface integrals of vector fields. Curl and divergence. Differential forms. Theorems of Green and Gauss; general Stokes' theorem. After MATH 120, and 222 or 225 or equivalent. QR

MATH 251b / EENG 434b / ENAS 496b / S&DS 351b, Stochastic Processes Yihong Wu and Sahand Negahban
Introduction to the study of random processes including linear prediction and Kalman filtering, Poison counting process and renewal processes, Markov chains, branching processes, birth-death processes, Markov random fields, martingales, and random walks. Applications chosen from communications, networking, image reconstruction, Bayesian statistics, finance, probabilistic analysis of algorithms, and genetics and evolution. Prerequisite: S&DS 241 or equivalent. QR

MATH 260a / AMTH 260a, Basic Analysis in Function Spaces Kirill Serkh
Diagonalization of linear operators, with applications in physics and engineering; calculus of variations; data analysis. MATH 260 is a natural continuation of PHYS 301. Prerequisites: MATH 120, and 222 or 225. QR

MATH 270a, Set Theory Gregg Zuckerman
Algebra of sets; finite, countable, and uncountable sets. Cardinal numbers and cardinal arithmetic. Order types and ordinal numbers. The axiom of choice and the well-ordering theorem. After MATH 120 or equivalent. QR

MATH 300b, Topics in Analysis Wilhelm Schlag
An introduction to analysis, with topics chosen from infinite series, the theory of metric spaces, and fixed-point theorems with applications. Students who have taken MATH 230, 231 should take MATH 301 instead of this course. After MATH 250 or with permission of instructor. QR

* **MATH 301a, Introduction to Analysis** Peter Jones
Foundations of real analysis, including metric spaces and point set topology, infinite series, and function spaces. After MATH 230, 231 or equivalent. QR

MATH 305b, Real Analysis Hee Oh
The Lebesgue integral, Fourier series, applications to differential equations. After MATH 301 or with permission of instructor. QR

MATH 310a, Introduction to Complex Analysis Ronald Coifman
An introduction to the theory and applications of functions of a complex variable. Differentiability of complex functions. Complex integration and Cauchy's theorem. Series expansions. Calculus of residues. Conformal mapping. After MATH 231 or 250 or equivalent. QR

* **MATH 315b, Intermediate Complex Analysis** Richard Beals
Continuation of MATH 310. Topics may include argument principle, Rouché's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem. Functions of finite order, Hadamard's theorem, meromorphic functions. Mittag-Leffler's theorem, subharmonic functions. After MATH 310. QR RP

* **MATH 320a, Measure Theory and Integration** Arie Levit
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals. After MATH 305 or equivalent. QR RP

* **MATH 325b, Introduction to Functional Analysis** Jeremy Hoskins
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 320. QR RP

MATH 330b / S&DS 400b, Advanced Probability Sekhar Tatikonda
Measure theoretic probability, conditioning, laws of large numbers, convergence in distribution, characteristic functions, central limit theorems, martingales. Some knowledge of real analysis assumed. QR

* **MATH 345a, Modern Combinatorics** Mathias Schacht
Recent developments and important questions in combinatorics. Relations to other areas of mathematics such as analysis, probability, and number theory. Topics include probabilistic method, random graphs, random matrices, pseudorandomness in graph theory and number theory, Szemeredi's theorem and lemma, and Green-Tao's theorem. Prerequisite: MATH 244. QR

MATH 350a, Introduction to Abstract Algebra Marketa Havlickova
Group theory, structure of Abelian groups, and applications to number theory. Symmetric groups and linear groups including orthogonal and unitary groups; properties of Euclidean and Hermitian spaces. Some examples of group representations. Modules over Euclidean rings, Jordan and rational canonical forms of a linear transformation. After MATH 222 or equivalent. QR

MATH 353a, Introduction to Representation Theory Oleksandr Tsymbaliuk
An introduction to basic ideas and methods of representation theory of finite groups and Lie groups. Examples include permutation groups and general linear groups. Connections with symmetric functions, geometry, and physics. After MATH 222 or equivalent.

* **MATH 354b, Number Theory** Ross Berkowitz
Prime numbers; quadratic reciprocity law, Gauss sums; finite fields, equations over finite fields; zeta functions. After MATH 350. QR

MATH 360a, Introduction to Lie Groups PhilSang Yoo
Lie groups as the embodiment of the idea of continuous symmetry. The exponential map on matrices and applications; spectral theory; examples and structure of Lie groups and Lie algebras; connections with geometry and physics. After MATH 231 or 250 or equivalent. MATH 300 or 301 and MATH 350 recommended. QR

MATH 370b, Fields and Galois Theory Asher Auel
Rings, with emphasis on integral domains and polynomial rings. The theory of fields and Galois theory, including finite fields, solvability of equations by radicals, and the fundamental theorem of algebra. Quadratic forms. After MATH 350. QR

MATH 373b, Algebraic Number Theory Asher Auel
Structure of fields of algebraic numbers (solutions of polynomial equations with integer coefficients) and their rings of integers; prime decomposition of ideals and finiteness of the ideal class group; completions and ramification; adeles and ideles; zeta functions. Prerequisites: MATH 310 and 370. QR

MATH 380a, Modern Algebra I Yifeng Liu
A survey of algebraic constructions and theories at a sophisticated level. Topics include categorical language, free groups and other free objects in categories, general theory of rings and modules, artinian rings, and introduction to homological algebra. After MATH 350 and 370. QR RP

MATH 381b, Modern Algebra II Kalina Mincheva
This course is the noncommutative counterpart to MATH 380. Abstract and concrete groups, rings and fields play a fundamental role. The main new concept is the notion of a left (right) module over a possibly noncommutative ring. The category R-mod of all left modules over a ring R encodes important information about the isomorphism class of R. After MATH 380. QR RP

MATH 435b, Differential Geometry Vincent Moncrief
Applications of calculus to the study of the geometry of curves and surfaces in Euclidean space, intrinsic differential geometric properties of manifolds, and connections with non-Euclidean geometries and topology. After MATH 231 or 250 or equivalent. QR

MATH 440a, Introduction to Algebraic Geometry Kalina Mincheva
Students develop the theory of algebraic varieties, which are zero sets of polynomial equations, starting with basic commutative algebra and include Groebner basis, chain conditions, and the ideal membership problem. Additional varieties include affine, projective, quasi-projective. We prove Hilbert's Nullstellensatz (one of the most important theorems of classical algebraic geometry) and discuss different notions of dimension and how they relate. We define maps between varieties, morphisms, rational, and birational maps. The remaining

topics are (but are not limited to) singularity theory, normalization and blowups, elimination theory, resultants, and divisors on algebraic varieties. Some focus on computational aspects of algebraic geometry is included. Prerequisite: MATH 350 or equivalent. QR

MATH 470a or b, Individual Studies Staff
Individual investigation of an area of mathematics outside of those covered in regular courses, involving directed reading, discussion, and either papers or an examination. A written plan of study approved by the student's adviser and the director of undergraduate studies is required. The course may normally be elected for only one term.

MATH 475a or b, Senior Essay Staff
Highly qualified students may write a senior essay under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the director of undergraduate studies early in the fall term.

* **MATH 480a or b, Senior Seminar: Mathematical Topics** Staff
A number of mathematical topics are chosen each term—e.g., differential topology, Lie algebras, mathematical methods in physics—and explored in one section of the seminar. Students present several talks on the chosen topic. One section each year is devoted to topics of interest to Economics and Mathematics majors, and is co-taught by a member of the Economics department.

OTHER COURSES RELATED TO MATHEMATICS

CPSC 365b, Algorithms James Glenn
Paradigms for algorithmic problem solving: greedy algorithms, divide and conquer, dynamic programming, and network flow. NP completeness and approximation algorithms for NP-complete problems. Algorithms for problems from economics, scheduling, network design and navigation, geometry, biology, and optimization. Provides algorithmic background essential to further study of computer science. Either CPSC 365 or CPSC 366 may be taken for credit. Prerequisites: CPSC 202 and 223. QR

PHIL 267a, Mathematical Logic Sun-Joo Shin
An introduction to the metatheory of first-order logic, up to and including the completeness theorem for the first-order calculus. Introduction to the basic concepts of set theory. Prerequisite: PHIL 115 or permission of instructor. QR

* **PHIL 427b, Computability and Logic** Sun-Joo Shin
A technical exposition of Gödel's first and second incompleteness theorems and of some of their consequences in proof theory and model theory, such as Löb's theorem, Tarski's undefinability of truth, provability logic, and nonstandard models of arithmetic. Prerequisite: PHIL 267 or permission of instructor. QR, HU