MATHEMATICS

See also Applied Mathematics.

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Mathematics has many aspects: it is the language and tool of the sciences, a cultural phenomenon with a rich historical tradition, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics major provides a broad education in various areas of mathematics in a program flexible enough to accommodate many ranges of interest.

PREREQUISITE
The prerequisite for both the B.A and B.S. degree programs is calculus through the level of multivariable calculus, by completion of MATH 120 or the equivalent. Completion of MATH 230 and 231 waives all prerequisite requirements.

PLACEMENT PROCEDURES
The department offers a three-term sequence in calculus, MATH 112, 115, and 120. Students who have not taken calculus at Yale and who wish to enroll in calculus must take the mathematics online placement examination; a link to the online examination and additional information are available on the department website. A calculus advising session will be held at the beginning of the fall term to answer student questions about placement.

MATH 112 is an introductory course that presupposes basic skills in high school algebra, geometry, and trigonometry. Enrolling students are expected to know the basic definitions of the trigonometric functions, synthetic division, factorization, and elementary area and volume formulas of plane and solid geometry. MATH 115 presupposes familiarity with the topics covered in MATH 112. MATH 120 presupposes familiarity with the topics covered in MATH 115.

MATH 230, 231 is an advanced course sequence in linear algebra and introductory analysis for students with exceptionally strong backgrounds in mathematics. Students who wish to enroll in MATH 230 should consult with the instructor of the course. After MATH 115, students with a strong interest in abstract mathematics should consider taking MATH 230, 231.

REQUIREMENTS OF THE MAJOR

B.A. degree program The B.A. degree program normally consists of ten term courses in Mathematics numbered 222 or higher, including the senior requirement (MATH 475 or 480); excluding, however, MATH 470. At most two courses from the introductory sequences (MATH 222, 225, 230, 231, 250) can be counted toward the major. Each student is expected to take vector calculus and linear algebra: either MATH 230 and 231, or either one of MATH 222 or 225 with MATH 250. MATH 225 is strongly recommended over MATH 222, for providing an introduction to proof writing, which is an essential skill in upper level mathematics courses. To acquire both depth and breadth in the field, students are required to take at least two term courses in each of three of the following five categories: analysis; algebra and number theory; statistics and applied mathematics; geometry and topology; and logic and foundations.

B.S. degree program A candidate for the B.S. degree must take at least two advanced term courses in the physical sciences, such as CHEM 333, 470, or PHYS 401, 402, in addition to the ten term courses required for the B.A. Such courses require the approval of the director of undergraduate studies (DUS); written approval is advised.

Both B.A. and B.S. degree programs Each major program must also include at least one course in at least two of the three core areas: real analysis; algebra; and complex analysis. Taking courses from all three core areas is strongly recommended.

Distinction in the major To be eligible for Distinction in the Major, a student must have completed at least one course from each of the three core areas. The categories and core areas to which each course belongs are indicated in the course listings.

The intensive major Candidates for a degree with an intensive major in Mathematics must take courses in all three of the core areas: real analysis; algebra; and complex analysis. Intensive majors are also expected to include at least two graduate term courses in the Mathematics department, or equivalent independent study, in their programs. Familiarity with the material of the following courses is prerequisite to graduate courses in each category: algebra: two courses between 350 and 399; analysis: MATH 301, 305, 310; algebraic topology: MATH 301, 350; logic and foundations: MATH 270.

Substitutions With permission of the Math DUS, up to two courses from other departments may be counted towards the required courses. For a list of courses that are typically approved, visit the FAQ page on the Math department website.

Credit/D/Fail Courses taken Credit/D/Fail may not be counted toward the requirements of the major.

Roadmap See visual roadmap of the requirements.
SENIOR REQUIREMENT

During the senior year students majoring in Mathematics normally take the senior seminar (MATH 480). Alternatively, with the consent of the DUS, highly qualified students may write a senior essay in MATH 475 under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the DUS early in the fall term.

ADVISING

Students interested in pursuing further study in pure mathematics should include MATH 301, 305, 310, 350, 370, and 430 in their programs, and should consider taking one or more graduate-level courses. Students interested in applications of mathematics should include MATH 300 or 301, 310, 350, and a selection of courses from MATH 241, 242, 244, 246, 251, 260, and CPSC 440.

Courses related to mathematics Each Mathematics major is urged to acquire additional familiarity with the uses of mathematics by taking courses in Applied Mathematics, Computer Science, Engineering and Applied Science, Economics, Philosophy, Physics, Statistics & Data Science, or other departments. In some instances a limited number of such courses may be counted among the ten courses required for the major in Mathematics, with the approval of the DUS.

Graduate work Each year the Mathematics and Statistics & Data Science departments offer a large number of graduate courses, some of which are accessible to undergraduates with advanced preparation in mathematics. Further information may be obtained from the DUSes, whose permission, with that of the relevant director of graduate studies, is required for admission.

Combined B.S./M.S. degree program Students who, by the end of their senior year, complete the requirements of the department for the M.S. in Mathematics are eligible to receive this degree at their Senior Commencement. Required are: (1) eight term courses numbered 500 or higher, most of which must be completed with grades of B or better; (2) passing a written qualifying examination of the student’s choice from analysis, algebra, or topology.

The master’s program is in no sense a substitute for the B.A. or B.S. program; rather, it is designed to accommodate a very few exceptional students who, by means of accelerated or independent study, can satisfy the department as to their command of the content of the normal undergraduate program by the end of the junior year. Candidates must submit a proposal that foresees this level of achievement to the DUS no later than the last day of classes in their fifth term of enrollment in Yale College. If approved by the department, the proposal is forwarded to the Yale College Dean’s Office. Students’ status and progress are reviewed before they are permitted to continue in the program in the senior year. For more information on Yale College requirements for the program, see Section K, Special Arrangements, “Simultaneous Award of the Bachelor’s and Master’s Degrees,” in the Academic Regulations.

Students take at least two graduate term courses in the junior year (normally courses in algebra or analysis are the first graduate courses taken). The general oral examination covers a list of topics available from the director of graduate studies and is accepted in lieu of the usual senior oral presentation. Details concerning the requirements for the master’s degree may be obtained from the director of graduate studies.

REQUIREMENTS OF THE MAJOR

Prerequisite MATH 120 or equivalent; completion of MATH 230 and 231; waive all prerequisite requirements

Number of courses B.A. – 10 term courses numbered 222 or higher (incl senior essay), excludes Math 470; B.S. – same, with 2 addtl adv courses in physical sciences approved by DUS

Specific courses required B.A. and B.S. – MATH 310 and 231; or MATH 222 or 225 with MATH 250

Distribution of courses B.A. and B.S. – 2 courses in each of 3 categories chosen from: analysis; algebra and number theory; stat and applied math; geometry and topology; logic and foundations; 1 course from 2 of 3 core areas chosen from: real analysis; algebra; and complex analysis

Substitution permitted With DUS permission, up to 2 courses from other depts as specified

Intensive major Courses in all 3 core areas; 2 MATH grad courses or equivalent independent study counted among the required courses

Senior requirement Senior sem (MATH 480) or, with DUS permission, senior essay (MATH 475) and oral report

Mathematics has many aspects. It is the language and tool of the sciences, a key part of cultural development since ancient times, and a model of abstract reasoning. The course offerings and the major in Mathematics reflect these multiple facets. The Mathematics program provides a broad education in various areas of mathematics and is flexible enough to accommodate many interests. Mathematics majors have numerous options after graduation, including graduate study in mathematics or in various fields of application, government or corporate laboratory work, consulting, finance and banking, and teaching. Students considering a major in Mathematics are encouraged to consult with the director of undergraduate studies (DUS) during their first year.

A variety of resources are available to students who want additional help in any mathematics course:

• Each course instructor holds office hours for students multiple times a week.
• Undergraduate peer tutors and learning assistants associated with many of the courses provide help through regular office hours.
• The Mathematics department offers coaching sessions for students in calculus courses.
• Drop-in tutoring with Math & Science Tutors is available in most residential colleges and at the Poorvu Center for Teaching and Learning. For details, see the Poorvu Center STEM Tutoring & Programs web page.
COURSES
A full description of offered mathematics courses can be found in Yale Course Search. The department offers several courses that satisfy the QR distributional requirement and do not assume knowledge of mathematics beyond the basic high-school level. They include:

- MATH 106, The Shape of Space
- MATH 107, Mathematics in the Real World
- MATH 108, Estimation and Error

The courses below include the ones most frequently taken by first-year students. Enrollment requires appropriate placement based on the online placement exam.

- MATH 110 (fall) and MATH 111 (spring) form a two-term introduction to differentiation, in which precalculus and calculus topics are combined. Successful completion of MATH 110 and 111 is equivalent to MATH 112 in that it satisfies the same major and professional-school (e.g. premed) requirements; however, a student completing MATH 110 and 111 receives two course credits and two QR credits. Only students who complete MATH 110 will be allowed to enroll in MATH 111. No prior exposure is assumed; some knowledge of algebra and precalculus mathematics is helpful.
- MATH 112 is the introductory course for students with a strong foundation in high school mathematics. It focuses primarily on differentiation and assumes no previous exposure to calculus.
- MATH 115 builds on MATH 112 and focuses primarily on integration and infinite series. Knowledge of topics covered in MATH 112 is assumed.
- MATH 120 combines the ideas and techniques of one-variable calculus with vector geometry and algebra to deal with geometrical situations in planes and in three-dimensional space. Knowledge of topics covered in MATH 112 and MATH 115 or MATH 116 is assumed.

The courses below serve as options after MATH 112 for calculus students who have a specific focus. Students interested in the biological sciences should consider MATH 116 (fall) and/or MATH 121 (spring), those interested in economics should consider MATH 118.

- MATH 116 is designed for bioscience and premedical students. It combines differential equations with modeling and applications in biology. Knowledge of topics covered in MATH 112 is assumed.
- MATH 118 is designed for students interested in economics and social sciences who do not intend to take additional mathematics courses. It covers basic ideas in linear algebra as well as differential calculus of several variables. Knowledge of topics covered in MATH 112 is assumed.
- MATH 121 is designed for bioscience and premedical students. It combines multivariable calculus with applications in biology. Knowledge of topics covered in MATH 115 or MATH 116 is assumed.

The higher-level courses below assume at least a year of calculus and are available to unusually well prepared first-year students.

- MATH 222 and MATH 225 deal with linear algebra, the common language for a wide variety of applications involving many variables. MATH 222 emphasizes computations and applications of linear algebra, while MATH 225 focuses on geometric and conceptual issues and the logical structure of the subject. Students normally take linear algebra after completing MATH 120; however, students who are sufficiently prepared and motivated may take MATH 222 or MATH 225 concurrently with MATH 120.
- MATH 230 and MATH 231 is a demanding, two-term course sequence on calculus of many variables, designed for students with a firm grasp of one-variable calculus and a strong interest in mathematics. It emphasizes conceptual and logical structure and pays considerable attention to proofs and challenging problems. This course covers with greater rigor the material studied in MATH 120, MATH 225, and MATH 250, but assumes no prior knowledge of those topics.
- MATH 250 is a rigorous introduction to calculus of many variables, based on linear algebra. It focuses on proofs of calculus results and their generalizations to higher dimensions; knowledge of topics covered in MATH 120, and MATH 222 or MATH 225 is assumed. The sequence of MATH 120, MATH 222 or MATH 225, and MATH 250 serves as an alternative to MATH 230 and MATH 231 for the requirements of the Mathematics major.

Additional higher-level courses may be open to exceptionally well prepared first-year students. Interested students should seek advice from the DUS or another knowledgeable faculty member.

FACULTY OF THE DEPARTMENT OF MATHEMATICS

Professors Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Richard Kenyon, Yifeng Liu, Ivan Losev, Gregory Margulis, Yair Minsky, Vincent Moncrief, Andrew Neitzke, Hee Oh, †Nicholas Read, Vladimir Rokhlin, Wilhelm Schlag, George Seligman (Emeritus), ‡Daniel Spielman, Van Vu, †John S. Wettlaufer, Gregg Zuckerman

J. W. Gibbs Assistant Professors Yariv Aizenbud, Paul Apisa, Subhajit Dey, Gurbir Dhillon, Samuel Edwards, Ariel Jaffe, Boris Landa, Or Landesberg, Arie Levit, Ofir Lindenbaum, Gal Mishne, Franco Vargas Pallete, Cosmin Pohoata, Fei Qi, Congling Qiu, Kirill Serkh, Caglar Uyanik, Tom VandenBoom, Anibal Velozo
**Adjunct Professors** Michael Goldstein, Gil Kalai, Alex Lubotzky, Jacques Peyriere, Mathias Schacht

**Senior Lecturers** John Hall, Miki Havlickova

**Lecturers** Ian Adelstein, Asher Auel, James Barnes, Robert McDonald, Itziar Ochoa de Alaiza Gracia, Erik Rosenthal, Brett Smith, Sarah Vigliotta

†A joint appointment with primary affiliation in another department.

View Courses