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M.S., M.Phil., Ph.D.

Chair
Wilhelm Schlag

Director of Graduate Studies
Ivan Loseu

Professors Richard Beals (Emeritus), Jeffrey Brock, Andrew Casson (Emeritus), Ronald Coifman, Igor Frenkel, Howard Garland (Emeritus), Anna Gilbert, Alexander Goncharov, Roger Howe (Emeritus), Peter Jones, Richard Kenyon, Ivan Loseu, Alexander Lubotzky (Adjunct), Gregory Margulis (Emeritus), Yair Minsky, Vincent Moncrief (Physics), Andrew Neitzke, Hee Oh, Nicholas Read (Physics; Applied Physics), Vladimir Rokhlin (Computer Science), Wilhelm Schlag, John Schotland, George Seligman (Emeritus), Charles Smart, Daniel Spielman (Computer Science), Van Vu, Lu Wang, John Wettlaufer (Earth & Planetary Sciences; Physics), Gregg Zuckerman (Emeritus)

Assistant Professor Junliang Shen

FIELDS OF STUDY
Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; probability; Kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; combinatorics and discrete mathematics.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE
In order to qualify for the Mathematics Ph.D., all students are required to:

1. Complete eight term courses at the graduate level, at least two with Honors grades.
2. Pass qualifying examinations on their general mathematical knowledge;
3. Submit a dissertation prospectus;
4. Participate in the instruction of undergraduates;
5. Be in residence for at least three years;
6. Complete a dissertation that clearly advances understanding of the subject it considers.

All students must also complete any other Graduate School of Arts and Sciences degree requirements; see Degree Requirements under Policies and Regulations.

The normal time for completion of the Ph.D. program is five years. Requirement (1) normally includes basic courses in algebra, analysis, and topology. A sequence of
three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term. All qualifying examinations must be passed by the end of the second year. There is no limit to the number of times that students can take the exams, and so they are encouraged to take them as soon as possible.

The dissertation prospectus should be submitted during the third year.

The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1)–(5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 991, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

HONORS REQUIREMENT
Students must meet the Graduate School’s Honors requirement by the end of the fourth term of full-time study.

TEACHING
Teaching experience is integral to graduate education at Yale. Therefore, teaching is required of all graduate students, typically one term per year. Generally, first-year students work as coaches for calculus classes, meeting with small discussion sections of undergraduates. Second-year students often work as teaching assistants for a linear algebra class (MATH 222, MATH 225, or MATH 226), real analysis (MATH 255 or MATH 256), or discrete mathematics (MATH 244); duties usually include holding office hours or leading discussion sections.

In the spring of their second year, graduate students attend the Lang Teaching Seminar (MATH 827). In this lunch seminar, experienced faculty help students understand the challenges of teaching and prepare students to lead their own section of calculus in the following year and beyond.

Students who require additional support from the Graduate School after the fifth year of study must teach additional terms, if needed.

MASTER’S DEGREES

M.Phil. See Degree Requirements under Policies and Regulations.

M.S. Students who withdraw from the Ph.D. program may be eligible to receive the M.S. degree if they have met the requirements and have not already received the M.Phil. degree. For the M.S., students must successfully complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year.

COURSES

MATH 500a, Algebra  Ivan Loseu
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings such as localization and integral extension,
connections to algebraic geometry, categories, functors and functor morphisms, tensor product and Hom functors, and projective modules. Other topics may be discussed at the instructor's discretion. Prerequisites: MATH 350 and MATH 370.

MATH 515b, Intermediate Complex Analysis  
Ebru Toprak
Topics may include argument principle, Rouché's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem; functions of finite order, Hadamard's theorem, meromorphic functions; Mittag-Leffler's theorem, subharmonic functions.

MATH 520a, Measure Theory and Integration  
Or Landesberg
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 525b, Introduction to Functional Analysis  
Wilhelm Schlag
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 520.

MATH 526a, Introduction to Differentiable Manifolds  
Subhadip Dey
This is an introduction to the general theory of smooth manifolds, developing tools for use elsewhere in mathematics. A rough plan of topics (with the later ones as time permits) includes (1) manifolds, tangent spaces, vector fields and flows; (2) natural examples, submanifolds, quotient manifolds, fibrations, foliations; (3) vector and tensor bundles, differential forms; (4) Lie derivatives, Lie algebras and groups; (5) embedding, immersions and transversality; (6) Sard's theorem, degree and intersection. Prerequisites: some multivariable calculus, linear algebra, and topology.

MATH 543b / CPSC 543b, Optimal Transport: Theory, Algorithms, and Applications to Data Science  
Smita Krishnaswamy
Optimal transport started with Gaspar Monge in the 1700s when he stated the problem of moving a large pile of sand (whose shape is a probability distribution) to a target pile with minimal effort. The optimal transport plan not only gives a coupling between distributions but also a metric between such probability measures, which has found use in everything from modern neural networks to economic resource allocation problems, to shape matching in computer vision. This course covers the theoretical foundations as well as computational aspects of optimal transport starting with the original formulations as maps between discrete measures and extending to general measures as well as the key Kantorovich relaxation as a coupling between measures and its metric properties. We also cover algorithmic foundations of optimal transport using linear programs that have recently been sped-up via entropic regularizations. In addition to the primal form, we cover the dual form and relaxations which lead to integral probability metrics. We vary the ground space of optimal transport from Euclidean, to arbitrary metrics, to graphs. We move from static to dynamic formulations of optimal transport, which can provide paths of flow for dynamics that are energy-constrained. Finally, we cover important extensions such as unbalanced optimal transport which allows for transport between generic measures (without the same volume) and for Gromov-Wasserstein distances between measures on different metric spaces. Prerequisites: MATH 241, CPSC 202, CPSC 223, and CPSC 365. Knowledge of Python programming is also required.
MATH 544a, Introduction to Algebraic Topology  Sebastian Hurtado - Salazar
This is a one-term graduate introductory course in algebraic topology. We discuss
algebraic and combinatorial tools used by topologists to encode information about
topological spaces. Broadly speaking, we study the fundamental group of a space,
its homology, and its cohomology. While focusing on the basic properties of these
invariants, methods of computation, and many examples, we also see applications
toward proving classical results. These include the Brouwer fixed-point theorem, the
Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher's
Algebraic Topology, which is available for free on his website.

MATH 619a, Foundations of Algebraic Geometry  Sam Raskin
This course provides an introduction to the language of basic ideas of algebraic
geometry. We study affine and projective varieties, and introduce the more general
theory of schemes. Our main references are Robin Hartshorne's book and Ravi Vakil's
lecture notes. Prerequisite: commutative algebra at the level of MATH 500/501.

MATH 640a / AMTH 640a or b / CPSC 640a or b, Topics in Numerical Computation
Vladimir Rokhlin
This course discusses several areas of numerical computing that often cause difficulties
to non-numericists, from the ever-present issue of condition numbers and ill-posedness
to the algorithms of numerical linear algebra to the reliability of numerical software.
The course also provides a brief introduction to “fast” algorithms and their interactions
with modern hardware environments. The course is addressed to Computer Science
graduate students who do not necessarily specialize in numerical computation; it
assumes the understanding of calculus and linear algebra and familiarity with (or
willingness to learn) either C or FORTRAN. Its purpose is to prepare students for
using elementary numerical techniques when and if the need arises.

MATH 710a / AMTH 710a, Harmonic Analysis on Graphs and Applications to
Empirical Modeling  Ronald Coifman
The goal of this graduate-level class is to introduce analytic tools to enable the
systematic organization of geometry and analysis on subsets of RN (data). In
particular, extensions of multi-scale Fourier analysis on graphs and optimal graph
constructions for efficient computations are studied. Geometrization of various Neural
Net architectures and related challenges are discussed. Topics are driven by students
goals.

MATH 713a, Poisson Algebras and Poisson Geometry  Nicholas Ovenhouse
Poisson Geometry is, in some sense, a generalization of symplectic geometry and
is the formalism used to describe classical mechanics. We discuss basic definitions,
properties, and structural results about Poisson structures and look at many well-
known examples. Beyond the basic theory, more advanced topics may include: R-
matrix Poisson structures, integrable systems, Poisson structures on character varieties,
and connections to cluster algebras. Prerequisites are basic algebra and differential
gometry (such as in a first-year graduate course).

MATH 727a, Vertex Operator Algebras and Related Structures  Igor Frenkel
Vertex operator algebras (VOA) is an algebraic formulation of two-dimensional
conformal field theory. This course is dedicated to general theory of VOAs and
fundamental examples related to representation theory of affine Kac-Moody algebras,
Virasoro algebra, and the Monster group. Modular forms and functions is an important
class of structures that naturally appear in VOA theory. Various realizations of modular forms and functions also suggest the relation of VOA with the universal quantum Teichmüller space and a mathematical construction of three-dimensional quantum gravity.

**MATH 728a, Kleinian Groups and Dynamics**  Hee Oh
We discuss various topics on dynamics on hyperbolic manifolds.

**MATH 729a, Topics in Teichmüller Theory and Mapping Class Groups**  Yair Minsky
Surfaces and their geometric structures play roles throughout mathematics. Of particular interest in this course are aspects of low-dimensional topology and geometry, as well as geometric group theory, but complex analysis plays a role as well. Depending on participant and lecturer interest, we cover aspects of the “classical” theory (Thurston compactification for example), coarse geometry of mapping class groups (and perhaps generalization to hierarchical hyperbolicity), and perhaps assorted topics like the study of infinite-type surfaces and their mapping class groups.

**MATH 827b, Lang Teaching Seminar**  Brett Smith and Su Ji Hong
This course prepares graduate students for teaching calculus classes. It is a mix of theory and practice, with topics such as preparing classes, presenting new concepts, choosing examples, encouraging student participation, grading fairly and effectively, implementing active learning strategies, and giving and receiving feedback. Open only to mathematics graduate students in their second year.

**MATH 991a / CPSC 991a, Ethical Conduct of Research**  Inyoung Shin
This course forms a vital part of research ethics training, aiming to install moral research codes in graduate students of computer science, math, and applied math. By delving into case studies and real-life examples related to research misconduct, students grasp core ethical principles in research and academia. The course also offers an opportunity to explore the societal impacts of research in computer science, math, and applied math. This course is designed specifically for first-year graduate students in computer science, applied math, and math. Successful completion of the course necessitates in-person attendance on eight occasions; virtual participation does not fulfill this requirement. In cases where illness, job interviews, or unforeseen circumstances prevent attendance, makeup sessions are offered.  © Course cr