MATHEMATICS (MATH)

MATH 108a, Estimation and Error C.J. Argue
A problem-based investigation of basic mathematical principles and techniques that help make sense of the world. Estimation, order of magnitude, approximation and error, counting, units, scaling, measurement, variation, simple modeling. Applications to demographics, geology, ecology, finance, and other fields. Emphasis on both the practical and the philosophical implications of the mathematics. Permission of instructor required. Enrollment limited to 25 students who have not previously taken a high school or college calculus course. QR

* **MATH 110a, Introduction to Functions and Calculus I** John Hall
Comprehensive review of precalculus, limits, differentiation and the evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of MATH 110 and 111 is equivalent to MATH 112. No prior acquaintance with calculus is assumed; some knowledge of algebra and precalculus mathematics is helpful. The course includes mandatory weekly workshops, scheduled at the beginning of term. Placement into MATH 110 on the Mathematics placement exam is required. Enrollment in MATH 110 is through preference selection, except during April registration (where sections are open to everyone who has placement in the course). QR

* **MATH 111b, Introduction to Functions and Calculus II** John Hall
Continuation of MATH 110. Comprehensive review of precalculus, limits, differentiation and evaluation of definite integrals, with applications. Precalculus and calculus topics are integrated. Emphasis on conceptual understanding and problem solving. Successful completion of both MATH 110 and 111 is equivalent to MATH 112. The course includes mandatory weekly workshops, scheduled at the beginning of term. Prerequisite: MATH 110. Enrollment in MATH 111 is through preference selection. QR

* **MATH 112a or b, Calculus of Functions of One Variable I** Staff
This course introduces the notions of derivative and of definite integral for functions of one variable, with some of their physical and geometrical motivation and interpretations. Emphasis is placed on acquiring an understanding of the concepts that underlie the subject, and on the use of those concepts in problem solving. This course also focuses on strategies for problem solving, communication and logical reasoning. Placement into MATH 112 on the Mathematics placement exam is required. No prior acquaintance with calculus or computing assumed. May not be taken after MATH 111. Enrollment in MATH 112 is through preference selection, except during April registration (where sections are open to everyone who has placement in the course). QR

* **MATH 115a or b, Calculus of Functions of One Variable II** Staff
A continuation of MATH 112, this course develops concepts and skills at the foundation of the STEM disciplines. In particular, we introduce Riemann sums, integration strategies, series convergence, and Taylor polynomial approximation. We use these tools to measure lengths of parametric curves, areas of polar regions and volumes of solids of revolution, and we explore applications of calculus to other disciplines including physics, economics, and statistics. MATH 115 also focuses on strategies for

problem solving, communication, and logical reasoning. Prerequisite: MATH 111 or MATH 112, or placement into MATH 115 on the Mathematics placement exam. May not be taken after MATH 116. Enrollment in MATH 115 is through preference selection, except during April registration (in this case sections are open to everyone who has placement in the course). QR

* **MATH 116a, Mathematical Models in the Biosciences I: Calculus Techniques** John Hall

Techniques and applications of integration, approximation of functions by polynomials, modeling by differential equations. Introduction to topics in mathematical modeling that are applicable to biological systems. Discrete and continuous models of population, neural, and cardiac dynamics. Stability of fixed points and limit cycles of differential equations. Prerequisite: MATH 112, or placement into MATH 115/116 on the Mathematics placement exam. May not be taken after MATH 115. QR

* **MATH 118a or b, Introduction to Functions of Several Variables** Staff

A combination of linear algebra and differential calculus of several variables. Matrix representation of linear equations, Gauss elimination, vector spaces, independence, basis and dimension, projections, least squares approximation, and orthogonality. Three-dimensional geometry, functions of two and three variables, level curves and surfaces, partial derivatives, maxima and minima, and optimization. Intended for students in the social sciences, especially Economics. May not be taken after MATH 120 or 222. Prerequisite: MATH 112. QR

* **MATH 120a or b, Calculus of Functions of Several Variables** Staff

Analytic geometry in three dimensions, using vectors. Real-valued functions of two and three variables, partial derivatives, gradient and directional derivatives, level curves and surfaces, maxima and minima. Parametrized curves in space, motion in space, line integrals; applications. Multiple integrals, with applications. Divergence and curl. The theorems of Green, Stokes, and Gauss. Prerequisite: MATH 115 or 116, or placement into MATH 120 on the Mathematics placement exam. May not be taken after MATH 121. Enrollment in MATH 120 is through preference selection, except during April registration (where sections are open to everyone who has placement in the course). QR

MATH 222a or b / AMTH 222a or b, Linear Algebra with Applications Staff

Matrix representation of linear equations. Gauss elimination. Vector spaces. Linear independence, basis, and dimension. Orthogonality, projection, least squares approximation; orthogonalization and orthogonal bases. Extension to function spaces. Determinants. Eigenvalues and eigenvectors. Diagonalization. Difference equations and matrix differential equations. Symmetric and Hermitian matrices. Orthogonal and unitary transformations; similarity transformations. Students who plan to continue with upper level math courses should instead consider MATH 225 or 226. After MATH 115 or equivalent. May not be taken after MATH 225 or 226. QR

MATH 225a or b, Linear Algebra Staff

An introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, inner product spaces, spectral theorem. The course focuses on conceptual understanding and serves as an introduction to writing mathematical proofs. For an approach focused on applications rather than proofs, consider MATH 222. Students with a strong mathematical background or

interest are encouraged to consider MATH 226. Prerequisite: MATH 115 or equivalent. May not be taken after MATH 222, 226, or 231. QR

* **MATH 226a, Linear Algebra (Intensive)** Ebru Toprak
A fast-paced introduction to the theory of vector spaces, matrix theory and linear transformations, determinants, eigenvalues, inner product spaces, spectral theorem. Topics are covered at a deeper level than in MATH 225, and additional topics may be covered, for example canonical forms or the classical groups. The course focuses on conceptual understanding. Familiarity with writing mathematical proofs is recommended. For a less intensive course, consider MATH 225. For an approach focused on applications, consider MATH 222. Prerequisite: MATH 115 or equivalent. May not be taken after MATH 222, 225, or 231. QR

MATH 232b / AMTH 232b, Advanced Linear Algebra with Applications Ian Adelstein
This course is a natural continuation of MATH 222. The core content includes eigenvectors and the Spectral Theorem for real symmetric matrices; singular value decomposition (SVD) and principle component analysis (PCA); quadratic forms, Rayleigh quotients and generalized eigenvalues. We also consider a number of applications: optimization and stochastic gradient descent (SGD); eigen-decomposition and dimensionality reduction; graph Laplacians and data diffusion; neural networks and machine learning. A main theme of the course is using linear algebra to learn from data. Students complete (computational) projects on topics of their choosing. Prerequisites: MATH 120 and MATH 222, 225, or 226. This is not a proof-based course. May not be taken after MATH 340 (previously MATH 240. QR

MATH 241a / S&DS 241a, Probability Theory Yihong Wu
Introduction to probability theory. Topics include probability spaces, random variables, expectations and probabilities, conditional probability, independence, discrete and continuous distributions, central limit theorem, Markov chains, and probabilistic modeling. After or concurrently with MATH 120 or equivalent. QR

MATH 244a or b / AMTH 244a or b, Discrete Mathematics Staff
Basic concepts and results in discrete mathematics: graphs, trees, connectivity, Ramsey theorem, enumeration, binomial coefficients, Stirling numbers. Properties of finite set systems. Prerequisite: MATH 115 or equivalent. Some prior exposure to proofs is recommended (ex. MATH 225). QR

MATH 246a or b, Ordinary Differential Equations Staff
First-order equations, second-order equations, linear systems with constant coefficients. Numerical solution methods. Geometric and algebraic properties of differential equations. After MATH 120 or equivalent; after or concurrently with MATH 222 or 225 or 226 or equivalent. QR

MATH 247b / AMTH 247b, Intro to Partial Differential Equations Erik Hiltunen
Introduction to partial differential equations, wave equation, Laplace's equation, heat equation, method of characteristics, calculus of variations, series and transform methods, and numerical methods. Prerequisites: MATH 222 or 225 or 226, MATH 246 or ENAS 194 or equivalents. QR

MATH 255a or b, Analysis 1 Staff
Introduction to Analysis. Properties of real numbers, limits, convergence of sequences and series. Power series, Taylor series, and the classical functions. Differentiation and Integration. Metric spaces. The course focuses on conceptual

understanding. Familiarity with writing mathematical proofs is assumed, and is further developed in the course. Prerequisite: MATH 115 or equivalent, and MATH 225 or 226. May not be taken after MATH 256, 300, or 301. QR

* **MATH 256b, Analysis 1 (Intensive)** Staff
Fast-paced introduction to Analysis. Properties of real numbers, limits, convergence of sequences and series. Power series, Taylor series, and the classical functions. Differentiation and Integration. Metric spaces. The course focuses on conceptual understanding. Familiarity with writing mathematical proofs is assumed, and is further developed in the course. Prerequisite: MATH 115 or equivalent, and MATH 225 or 226. May not be taken after MATH 255, 300, or 301. QR

MATH 260b / AMTH 260b, Basic Analysis in Function Spaces Ronald Coifman
Diagonalization of linear operators, with applications in physics and engineering; calculus of variations; data analysis. MATH 260 is a natural continuation of PHYS 301. Prerequisites: MATH 120, and 222 or 225 or 226. QR

MATH 270a, Set Theory Charles Smart
Algebra of sets; finite, countable, and uncountable sets. Cardinal numbers and cardinal arithmetic. Order types and ordinal numbers. The axiom of choice and the well-ordering theorem. After MATH 120 or equivalent. QR

MATH 302a or b, Vector Analysis and Integration on Manifolds Staff
A rigorous treatment of the modern toolkit of multivariable calculus. Differentiation and integration in R^n. Inverse function theorem. Fubini's theorem. Multilinear algebra and differential forms. Manifolds in R^n. Generalized Stokes' Theorem. The course focuses on conceptual structure and proofs, and serves as a gateway to more advanced courses which use the language of manifolds. Prerequisites: MATH 225 or 226, and MATH 255 or 256. QR

MATH 305b, Analysis 2: Lebesgue Integration and Fourier Series Hee Oh
The Lebesgue integral, Fourier series, applications to differential equations. Prerequisites: MATH 225 or 226, and MATH 255 or 256 or 301. With permission of instructor, may be taken after MATH 225 or 226, and MATH 231 or 250. QR

MATH 310a, Introduction to Complex Analysis Richard Kenyon
An introduction to the theory and applications of functions of a complex variable. Differentiability of complex functions. Complex integration and Cauchy's theorem. Series expansions. Calculus of residues. Conformal mapping. Prerequisites: MATH 225 or 226 or 231, and MATH 255 or 256 or 230 or 250, and MATH 302 or 120. QR

* **MATH 315b, Intermediate Complex Analysis** Ebru Toprak
Continuation of MATH 310. Topics may include argument principle, Rouché's theorem, Hurwitz theorem, Runge's theorem, analytic continuation, Schwarz reflection principle, Jensen's formula, infinite products, Weierstrass theorem. Functions of finite order, Hadamard's theorem, meromorphic functions. Mittag-Leffler's theorem, subharmonic functions. After MATH 310. QR

* **MATH 320a, Measure Theory and Integration** Or Landesberg
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals. After MATH 305 or equivalent. QR

* **MATH 325b, Introduction to Functional Analysis** Wilhelm Schlag
Hilbert, normed, and Banach spaces; geometry of Hilbert space, Riesz-Fischer theorem; dual space; Hahn-Banach theorem; Riesz representation theorems; linear operators; Baire category theorem; uniform boundedness, open mapping, and closed graph theorems. After MATH 320, or after MATH 305 with permission of instructor. QR

MATH 330a / S&DS 400a, Advanced Probability Sekhar Tatikonda
Measure theoretic probability, conditioning, laws of large numbers, convergence in distribution, characteristic functions, central limit theorems, martingales. Some knowledge of real analysis assumed. QR

MATH 340b, Advanced Linear Algebra Staff
The course continues the study of linear algebra from MATH 225 or MATH 230/231. It discusses several aspects of linear algebra that are of crucial importance for the subject and its applications to abstract algebra, geometry and number theory. Topics include generalized eigenspaces and Jordan normal form theorem, dual vector spaces, bilinear and hermitian forms, symmetric and hermitian operators, Hom spaces and tensor products. Previously MATH 240. After MATH 225 or 226 or 231. Two semesters of proof-based mathematics courses are recommended.

* **MATH 345a, Modern Combinatorics** Van Vu
Recent developments and important questions in combinatorics. Relations to other areas of mathematics such as analysis, probability, and number theory. Topics include probabilistic method, random graphs, random matrices, pseudorandomness in graph theory and number theory, Szemeredi's theorem and lemma, and Green-Tao's theorem. Prerequisite: MATH 244. QR

MATH 350a or b, Introduction to Abstract Algebra Staff
Group theory, structure of Abelian groups, and applications to number theory. Symmetric groups and linear groups including orthogonal and unitary groups; properties of Euclidean and Hermitian spaces. Some examples of group representations. Modules over Euclidean rings, Jordan and rational canonical forms of a linear transformation. Prerequisites: one term of linear algebra and two terms of proof-based mathematics courses. (For example, MATH 225 and 255, or MATH 225 and 244, or MATH 230 and 231, or MATH 225 and 250.) QR

MATH 370b, Fields and Galois Theory Miki Havlickova
Rings, with emphasis on integral domains and polynomial rings. The theory of fields and Galois theory, including finite fields, solvability of equations by radicals, and the fundamental theorem of algebra. Quadratic forms. After MATH 350. QR

MATH 380a, Algebra Ivan Loseu
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings, such as localization and integral extension, connections to algebraic geometry, categories, functors and functor morphisms, tensor product and Hom functors, projective modules. Other topics may be discussed at instructor's discretion. After MATH 350 and 370. QR

MATH 421a / AMTH 420a, The Mathematics of Data Science Kevin O'Neill
This course aims to be an introduction to the mathematical background that underlies modern data science. The emphasis is on the mathematics but occasional applications

are discussed (in particular, no programming skills are required). Covered material may include (but is not limited to) a rigorous treatment of tail bounds in probability, concentration inequalities, the Johnson-Lindenstrauss Lemma as well as fundamentals of random matrices, and spectral graph theory. Prerequisite: MATH 305. QR, SC

MATH 435b, Differential Geometry Franco Vargas Pallete
Applications of calculus to the study of the geometry of curves and surfaces in Euclidean space, intrinsic differential geometric properties of manifolds, and connections with non-Euclidean geometries and topology. Prerequisites: MATH 225 or 226 or 231, and MATH 255 or 256 or 230 or 250, and MATH 302 or permission of instructor. QR

MATH 440b, Introduction to Algebraic Geometry Alexander Goncharov
Algebraic geometry is the study of algebraic varieties, which are the spaces described by zero sets of polynomial equations. This course is an introduction to algebraic geometry with a focus on algebraic curves. These are 1-dimensional varieties, which can also be viewed as Riemann surfaces, lying at the crossroads of many branches of mathematics. We develop the theory of algebraic curves including divisors, Hurwitz's theorem, Riemann-Roch theorem, Jacobians, and Abel-Jacobi theory. We also discuss some aspects of higher dimensional varieties. Prerequisites: MATH 310 and MATH 350. QR

MATH 470a or b, Individual Studies Miki Havlickova
Individual investigation of an area of mathematics outside of those covered in regular courses, involving directed reading, discussion, and either papers or an examination. A written plan of study approved by the student's adviser and the director of undergraduate studies is required. The course may normally be elected for only one term.

MATH 475a or b, Senior Essay Miki Havlickova
Interested students may write a senior essay under the guidance of a faculty member, and give an oral report to the department. Students wishing to write a senior essay should consult the director of undergraduate studies at least one semester in advance of the semester in which they plan to write the essay.

* **MATH 480a or b, Senior Seminar: Mathematical Topics** Staff
A number of mathematical topics are chosen each term—e.g., differential topology, Lie algebras, mathematical methods in physics—and explored in one section of the seminar. Students give several presentations on the chosen topic. Available for credit only to seniors majoring in Mathematics, Economics and Mathematics, or Mathematics and Philosophy. May not be taken after MATH 481.

* **MATH 481b, Senior Seminar: Topics in Economics and Mathematics** Kevin O'Neill and Dirk Bergemann
A number of topics at the intersection of economics and mathematics are chosen each term—e.g., the theory of networks, market design and equilibrium, information economics and probability —and explored in the seminar. Students present several talks on the chosen topic. This section is devoted to topics of interest to majors in Economics or Mathematics majors, and in particular to students in the joint major Economics and Mathematics. The seminar is co-taught by a member of the Economics Department. Available for credit only to seniors majoring in Mathematics, Economics and Mathematics, or Mathematics and Philosophy. May not be taken after MATH 480.